Wireless Communication Networks Between Distributed Autonomous Systems Using Self-Tuning Extremum Control

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Milestones

- Motivation and Issues
- Comms Propagation Modeling
- Self-Tuning Extremum Control
- Flight Test Results
Sensor Networks with Multiple UAS

Applications

- Nature Monitoring - Civil (Disaster, Forest Fire, Weather)
- Surveillance & Coverage - Military (SA, Decision Support, ISR)
- Remote Sensing - Science (GIS, Ocean Map Building, etc)
Research Goals

Dispatch a swarm of networked UAVs as communication relay nodes for real-time decision-making support and situational awareness
Research Goals & Issues

Research Issues

- High Bandwidth Communication Links (Max. Throughputs)
- Wide Area/Range Coverage (Network Coverage Control)
- Long-Term Communication Relay (Aerial Platforms)
Objective and Approach

- Develop control algorithms that allow UAVs to reposition themselves autonomously at optimal flight location to maximize the communications link quality.
Control Method

- Methods for controlling flying platforms to operate continually at the maximum point of a performance function can be termed real-time optimization or extremum control.
Real-Time Optimization

- Cost Function: Communication performance
- Constraint: UAV positioning equation

\[
\max_{x_k \in D} J_k(x_k) \quad \text{subject to} \quad x_{k+1} = f(x_k, u_k)
\]

- Cost Function (J)

\[
J(x_k) = J(x_k, y_k, z_k, \phi_k, x_{node,i})
\]

\[x_{node,i} = \text{communications nodes } (x_k, y_k, z_k, \phi_k) = \text{UAV position and attitude (bank)}\]

- Equations of 3D/2D UAV Motion

\[
f(x_k) : \begin{cases} 
x_{k+1} = x_k + v \cos(\psi_h) \Delta t \\
y_{k+1} = y_k + v \sin(\psi_h) \Delta t
\end{cases}
\]

where \( v \) is body-axis speed and \( \psi_h \) is the yaw angle of the vehicle.
Real-Time Optimization

- If partial derivatives of the cost function are known

Solution: Extremum Control (Gauss-Newton Optimization)

$$x_{k+1} = x_k + u_k = x_k - \alpha_k H_k^{-1} (x_k) \nabla J(x_k)$$

where

$$H_k = h_{ij}(x_k) = \frac{\partial^2 J}{\partial x_i \partial x_j}(x_k), \quad \nabla J(x_k) = \left( \frac{\partial J}{\partial x_1}(x_k), \ldots, \frac{\partial J}{\partial x_n}(x_k) \right)^T$$

Issue: 3-D Complex Optimization Problem

$$J(x_k, y_k, z_k, \phi_k, x_{node,i}) = J(\phi_k, ||d||)$$

where

$$||d|| = \sqrt{(x_{uav} - x_{node})^2 + (y_{uav} - y_{node})^2 + (z_{uav} - z_{node})^2}$$
Methodology

- **Gradient-Type Extremum Control**
  - Measured SNR is discontinuous and slow (1 Hz)
  - Subjective to noise and cluttered environment
  - Affected by the orientation of a UAV (fast maneuver)

  ✓ Computation of gradient/hessian values is nontrivial

- **Approaches and Solutions**
  - **Mathematical Communications Modeling**
    - Provide continuous reference values at fast mode
    - Predict a maximum operation point
  - **Model-Free Adaptive Extremum Control**
    - Gradient is obtained by numerical method without model
    - Robust to noise and cluttered environment
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Why Signal-to-Noise Ratio Model

\[ C = W \log_2 (1 + SNR) \]

: Shannon-Hartley Theorem

where \( C \) is channel capacity (bits per second) \( W \) - bandwidth (Hz) of the channel

✓ Channel capacity (\( C \)) is proportional to the SNR and the bandwidth (\( W \))

Signal-to-Noise Ratio (SNR) Model

\[
SNR(dBm) = \frac{P_r(dBm)}{P_n(dBm)} = \left( \frac{\lambda}{4\pi \|d\|} \right)^2 \frac{G_t G_r}{L_{ap}}
\]

where \( P_r(dBm) \) is the receiver power \( P_n(dBm) \) is noise power (-95 dBm) 
\( G_r(dB) \) is receiver antenna gain \( G_t(dB) \) is transmitter antenna gain

\( \lambda = c / f \) where \( f \) is the transmission frequency \( c = 3 \times 10^8 \) m/s \( \|d\| = \) distance

\[ L_p(dB) \equiv (4\pi \|d\| / \lambda)^2 \text{is path loss} \]

\[ L_\phi(dB) \text{ is antenna pattern loss} \]
Model for UAV Orientation Effects

Antenna Pattern Loss : Function of Arrival Angle $\gamma_i(t)$

$$\gamma_i(t) = -\theta_i(t) - \phi(t) \sin (\varphi_i(t) - \psi(t))$$

which is the angle between the incident ray and horizontal wing of a UAV

$$\theta_i(t) = \tan^{-1} \left( \frac{z(t) - z_{node,i}}{\sqrt{(x(t) - x_{node,i})^2 + (y(t) - y_{node,i})^2}} \right)$$

$$\varphi_i(t) = \tan^{-1} \left( \frac{y(t) - y_{node,i}}{x(t) - x_{node,i}} \right)$$

$\phi(t)$ is the UAV bank angle  
$\psi(t)$ is the heading angle of the UAV  
$\varphi_i(t)$ is the bearing angle
- Static SNR Map in East-North-Up coordinates
  - Fixed altitude, heading & bank angle
  - Path loss, Antenna pattern loss
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Use on-line gradient estimation of SNR function to drive the set point to its max location

On-line estimator does not require a precise model
Perturbation Based Gradient Estimator

- The purpose is to make $\theta - \theta^*$ as small as possible, so that the output is driven to its minimum $J^*$

How It Works?

- Let $y = J(\theta)$ be a general mapping function
- Assume $\hat{\theta}$ be a current parameter
- Perturbation $a \sin wt$ around $\hat{\theta}$ leads to
\[
y = J(\hat{\theta} + a \sin wt) \approx J(\hat{\theta}) + a \frac{\partial J}{\partial \theta}_{\theta=\hat{\theta}} \sin wt
\]

Peak-Seeking Architecture (Stability Proof by Kristic, 2001)
- Applying high-pass filter (differentiator) gets rid of constant term and leads to
\[
y_H \approx a \left. \frac{\partial J}{\partial \theta} \right|_{\theta=\hat{\theta}} \sin wt
\]
Demodulating $y_H$ with $\sin(\omega t)$ divides the signal into a low-frequency signal and high-frequency signal

$$\zeta = \frac{1}{2} a \frac{\partial J}{\partial \theta}_{\theta = \hat{\theta}} - \frac{1}{2} a \frac{\partial J}{\partial \theta}_{\theta = \hat{\theta}} \cos 2\omega t$$

Applying low-pass filter (integrator) gets rid of the sinusoidal term and provides an estimate of the gradient of $J(\theta)$

$$y_L \approx \frac{1}{2} a \frac{\partial J}{\partial \theta}_{\theta = \hat{\theta}}$$

The estimated gradient can be expressed by the parameter change

$$\hat{\theta} = k \frac{1}{2} a \frac{\partial J}{\partial \theta}_{\theta = \hat{\theta}}$$

Denote $\tilde{\theta} = \hat{\theta} - \theta^*$ the convergence error, and taking a derivative of the errors leads to

$$\hat{\tilde{\theta}} = \dot{\hat{\theta}} \approx k \frac{1}{2} a J''(\theta^*) \tilde{\theta}$$

which become stable with a proper choice of the parameter, $a$ and $k$ i.e., $kaJ''(\theta^*) < 0$
How Self-Tuning Extremum Control Works?

Key idea is to integrate an on-line gradient estimator into an extremum control to get optimal location for UAVs.

Consider 2-D Motion in \{I\} Frame

\[ f(x_k) : \begin{cases} \dot{x}(t) = v(t) \cos(\psi_h(t)) \\ \dot{y}(t) = v(t) \sin(\psi_h(t)) \end{cases} \]

where \( v \) is body-axis speed and \( \psi_h \) is the yaw angle of the vehicle.

Motion with Constant Speed

\[ x(t) = v \cos(\psi_h(t)) = f_1(\psi_h(t), x_0) \]
\[ y(t) = v \sin(\psi_h(t)) = f_2(\psi_h(t), y_0) \]

where \( v = \text{const} \)
Then SNR function becomes an implicit function of heading angle

\[ J = \text{SNR}(x(t), y(t)) = \text{SNR}(x(\psi_h(t)), y(\psi_h(t))) = J(\psi_h(t)) \]

Gradient Descent Extremum Control is expressed by

\[ \psi_{k+1} = \psi_k + \alpha_k \nabla J_\psi \]

where \( \nabla J_\psi = \frac{\partial J}{\partial \psi} \in \mathbb{R} \)

Assume that SNR is a quadratic function

\[ J(\hat{\psi}(t)) = J^* + \frac{\mu}{2}(\hat{\psi}(t) - \psi^*)^2 + w(t) \]

\( \hat{\psi}(t) \) is the current heading angle estimate

\( J^* \) is the maximum attainable value of the cost function

\( w(t) \) is a zero-mean white noise

\( \mu \) is the sensitivity of the quadratic curve

\( \psi^* \) is the heading angle maximizing \( J \)
Adaptive Convergence Control

Adaptive Convergence Rate $\alpha_k$

Armijo-Wolfe Conditions

\[
\begin{align*}
J(x_k + \alpha_k d_k) &\leq J(x_k) + c_1 \alpha_k d_k^T \nabla J(x_k) \\
d_k^T \nabla J(x_k + \alpha_k d_k) &\geq c_2 d_k^T \nabla J(x_k)
\end{align*}
\]

where $0 < c_1 < c_2 < 1$

the Armijo condition that prevents steps that are too long
the Wolfe condition which restricts steps that are too short

Adaptive Convergence Control Law

\[
\alpha_{k+1} = \gamma \alpha_k, \text{ where } \begin{cases} 
0 < \gamma < 1, & \text{if } \Delta J_{k+1} > \tau_{tv} \\
\gamma \geq 1, & \text{else } \Delta J_{k+1} < \tau_{tv}
\end{cases}
\]

where

\[
\Delta J_{k+1} = J_{k+1} - J_k \text{ or } d(\nabla J_\psi(t)) / dt \leq \tau_{tv}
\]

$\tau_{tv}$: a specified threshold value

\[
u_{com}(t) = \begin{cases} 
\dot{\psi}_{com}(t) = \dot{\psi}_{ss} & \text{if } |\dot{\psi}_{com}(t) - \dot{\psi}_{ss}| = v / R_{ss} \leq \varepsilon_{ss} \\
\dot{\psi}_{com}(t) = \dot{\psi}_{ss} + \mu \gamma \alpha(t) \dot{\psi}(t) & \text{other}
\end{cases}
\]
Applying On-Line Gradient Estimator

\[ \nabla J_{\psi}(t) = \frac{\partial J(\dot{\psi}(t))}{\partial \dot{\psi}(t)} = \mu(\dot{\psi}(t) - \psi^*) , \quad \frac{d}{dt}(\nabla J_{\psi}(t)) = \mu(\dot{\psi}(t)) \]

Then the extremum controller is expressed by

\[ \dot{\psi}_{com}(t) = \frac{d\psi(t)}{dt} = \alpha(t) \frac{d}{dt}(\nabla J_{\psi}) \]

\[ = \mu \alpha(t) \hat{\psi}(t) \]

\( \alpha(t) \): step length along the direction \( \nabla J \)

Optimal value can be obtained by Armijo-Wolfe conditions

Orbit Circle Guidance at Final Steady-Stage

\[ u(t) = \begin{cases} 
\dot{\psi}_{com}(t) = \dot{\psi}_{ss} & \text{if } |\dot{\psi}_{com}(t) - \dot{\psi}_{ss}| = \frac{v}{R_{ss}} |\leq \varepsilon_{ss} \\
\dot{\psi}_{com}(t) = \dot{\psi}_{ss} + \mu \alpha(t) \hat{\psi}(t) & \text{other}
\end{cases} \]

\( \dot{\psi}_{ss} \) is introduced to guarantee that the UAV will orbit with a constant radius \( R_{ss} \) at the final stage

\( R_{ss} = \frac{v}{\dot{\psi}_{ss}} \): a final approach circle radius.
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Rapid Flight Test Design Keys

- Reduce development time
- Upgrade is flexible
- Convenience of high level programming
Model Verification Flight Test

3 dB Omni-Directional Antenna

9 dB Sector Antenna
SNR Model Verification with respect to UAV Trajectories
SNR Model Verification

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SNR Variation with respect to UAV Trajectories

SNR at Rascal’s (2D) Trajectory (01 Aug 08)

Start Point (Anti-clockwise)

End Point

GCS Position (East 0, North 0)

North (m)

East (m)

0 dB 10 dB 20 dB 25 dB 30 dB 35 dB

SNR Variation with respect to UAV Trajectories
Comparison with SNR Model

SNR Error Plots Between Real and Model Values
Flight Test (Nov. 20, 2008)

- Validate the designed onboard adaptive self-tuning controller & the communication models

Network Coverage Control using Extremum-Seeking Control
Flight Test Set-Up

Sensor Node Locations & Flight Setup in Camp Roberts
UAV Trajectory over SNR MAP

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(Movie)

UAV Trajectory Control for Max Communication Links (SNR)
Plot of UAV Trajectory over SNR Maps
Plot of SNR Errors Between Model and Observation Ones
Conclusions

- Communication Propagation Model
  - Communication propagation model was developed, which include the effects of the path loss, antenna pattern loss, and the orientation of aerial platforms
  - Proposed models were validated through real flight tests

- Self-Tuning Extremum Control for UAVs Location
  - On-line adaptive gradient estimator was integrated into an extremum control architecture
  - Proposed self-estimating extremum control is robust to even low signal-to-noise ratio signal
  - Effectiveness of the self-tuning optimizer was validated through real time flight tests

- Applicable for Decentralized Network Coverage Control