System Dynamics Structures for Modeling Lawmaking Processes

Raymond Madachy*

Department of Systems Engineering, Naval Postgraduate School, Monterey, CA, 93943

ABSTRACT

Modeling and simulation can help improve lawmaking processes. System dynamics is a simulation methodology for modeling continuous systems that provides a rich and integrative framework for investigating lawmaking process phenomena and inter-relationships from a holistic perspective. Structures for modeling these processes are provided as reusable building blocks. These structures and their behaviors are process patterns that frequently occur. Examples are shown assembling these recurring structures into larger models demonstrating behavior patterns of lawmaking processes including feedback loops. The behaviors are visualized as process trends over time.

This paper overviews: 1) basic system dynamics elements and their applied instances in lawmaking, 2) generic flow processes which are small microstructures comprised of a few elements serving as modeling molecules with characteristic behaviors, 3) infrastructures composed of several microstructures producing more complex behaviors, 4) flow chains which are infrastructures consisting of a sequence of levels and rates (stocks and flows) that are model portion backbones, and 5) introductory examples of lawmaking process structures.

Even small system dynamics models have been shown useful for understanding complex public policy issues, and thus well suited to assess specific laws and/or aspects of local, national and international lawmaking processes. The structures and applied examples are provided as open source models for the community to incorporate, adapt and apply for lawmaking.

Keywords: Lawmaking Processes, System Dynamics, Modeling and Simulation

INTRODUCTION AND BACKGROUND

A scientific approach to lawmaking has the potential to improve the efficiency of lawmaking processes, and the effectiveness of laws created. Modeling and simulation can be used for these purposes.

Applying science to lawmaking was proposed by Schrunk because traditional methods haven't produced laws that consistently solve societal problems (Schrunk, 2005). Modeling and simulation are successfully applied across many disparate fields to gain better process understanding, and lawmaking is a fruitful area for investigation.

This work applies simulation concepts to create model structures that can be used to 1) evaluate the lawmaking process, i.e. the steps taken to create laws including their order, and 2) assess laws before implementation on how well they will meet their goals and compare options. The latter consideration includes all intended and unintended consequences of law implementation.

System dynamics was developed by Forrester to improve organizational structures and processes (Forrester, 1968), from which this work is ultimately *The Science of Laws Journal*, Vol. 3, No.1, (2017): 12-22. © 2017 The Science of Laws Institute (www.scienceoflaws.org) *Author to whom all correspondence should be addressed (e-mail: rjmadach@nps.edu). derived. It has been applied across numerous fields and is a commonly used method for modeling continuous systems.

PROCEEDING

Many system dynamics applications have been developed which could be adapted for lawmaking processes. However, the modeling task may be difficult and time consuming for new or even experienced modelers.

This work helps fill the knowledge gap for the lawmaking domain, and make the modeling easier. It organizes system dynamics model structures and behaviors for lawmaking processes starting with elemental components, incorporating them into basic flow structures and building up to larger infrastructures.

The taxonomy and process representations provide generalized and adaptable "plug and play" components of varying complexity to build lawmaking process models.

The structures and their behaviors are process patterns that frequently occur. The recurring structures are model "building blocks" that can be reused. They provide a framework for understanding, modifying and creating system dynamics models regardless of experience. With access to reusable formulations that have been repeatedly proven, previous work can be understood easier and the structures incorporated into new models with minimal modification.

Previous work for classifying system dynamic structures was been done in (Hines, 2000), where relatively small

scale generic "modeling molecules" are described. Simulation packages often come with usage examples, such as (Richmond et al., 1990) which provides descriptions of common building blocks. Other work that has provided a comprehensive modeling taxonomy for a specific domain is in (Madachy, 2008) for software development processes.

A written law is a piece of code that requires internal consistency and completeness to meet the law's purpose(s). Thus laws are very similar to software. It is found that many modeling structures for software development processes have strong analogies in the lawmaking process domain.

SYSTEM DYNAMICS OVERVIEW

System dynamics provides a very rich modeling environment. It can incorporate many formulations including equations, graphs, tabular data or otherwise. Models are formulated using continuous quantities interconnected in loops of information feedback and circular causality. The quantities are expressed as levels (also stocks or accumulations), rates (also called flows) and information links representing the feedback loops. See Appendix A for the underlying mathematical formulation.

The system dynamics approach involves the following concepts:

– defining problems dynamically, in terms of graphs over time

- striving for an endogenous ("caused within") behavioral view of the significant dynamics of a system

- thinking of all real systems concepts as continuous quantities interconnected in information feedback loops and circular causality

– identifying independent levels in the system and their inflow and outflow rates

- formulating a model capable of reproducing the dynamic problem of concern by itself

– deriving understandings and applicable policy insights from the resulting model

– implementing changes resulting from model-based understandings and insights, which was Forrester's overall goal.

A major principle is that the dynamic behavior of a system is a consequence of its own structure. Given this, the structure of a system can be focused on in order to effect different behavior. Improvement of a process thus entails an understanding and modification of its structure. The structures of the as-is and to-be processes are represented in models.

The existence of process feedback is another underlying principle. Elements of a system dynamics model can interact through feedback loops, where a change in one variable affects other variables over time, which in turn affects the original variable. Understanding and taking advantage of feedback effects can provide high leverage.

Below is an overview of terminology related to system dynamics model structures and behavior:

Elements are the smallest individual pieces in a system dynamics model: levels, rates, sources/sinks, auxiliaries

and information connections. See Figure 1 for their visualizations.

Generic flow processes are small microstructures and their variations comprised of a few elements, and are sometimes called modeling molecules. They are the building blocks, or substructures from which larger structures are created and usually contain approximately 2-5 elements.

Infrastructures refer to larger structures that are composed of several microstructures, typically producing more complex behaviors.

Flow chains are infrastructures consisting of a sequence of levels and rates (stocks and flows) that often form a backbone of a model portion. They house the process entities that flow and accumulate over time, and have information connections to other model components through the rates.

This paper does not explicitly discuss archetypes in detail. They present lessons learned from dynamic systems with specific structure that produces characteristic modes of behavior. The structures and their resultant dynamic behaviors are also called patterns. Whereas molecules and larger structures are the model building blocks, archetypes interpret the generic structures and draw dynamic lessons from them. Senge discusses organizational archetypes based on simple causal loop diagrams in The Fifth Discipline (Senge, 1990).

MODEL STRUCTURES AND BEHAVIORS

Next is a review of the basic model elements, generic flows and infrastructures. Specific structures for lawmaking process models and some behavioral examples will be identified. All of the lawmaking process structures are derived from one or more generic structures. Each structure can be represented with a diagram, summary of critical equations, and behavioral output.

The generic flow processes, infrastructures and behaviors are extensive and thorough. Due to space limitations only a few are illustrated for simple lawmaking processes. This paper will instead present important flow chains to use for model backbones as partially filled skeletons. The reader is encouraged to read supplemental traditional references on the smaller general structures for system dynamics (Forrester, 1968), (Hines, 2000), (Madachy, 2008), (Sterman, 2000).

MODEL ELEMENTS FOR LAWMAKING

The basic elements of system dynamics models are levels, flows, sources/sinks, auxiliaries and connectors or feedback loops. Figure 1 serves as a legend showing the standard notation of these elements in a rate and level system with an auxiliary variable connected to the rate via an information link. Next the standard elements are briefly reviewed with example instantiations for lawmaking processes.



Figure 1. Model Notation of a Rate and Level System

Levels are the state variables representing system accumulations. Their counts can be measured in a real system at a snapshot of time (e.g. the number of current laws on the books). Typical state variables are laws or rights, violations, lawsuits, or the numbers of people involved in legal systems. These major level types are detailed further per the following:

- <u>Laws or Rights</u> These may include laws (e.g. statutes, ordinances, regulations, common laws); copyrights or intellectual property rights for any jurisdiction, etc. Laws can be represented at any stage in the lawmaking process from proposed bills to amended or repealed laws, and for any level of jurisdiction. Rights levels can be in different process stages from initial filing to infringement (see example flow chains in the Lawmaking Process Chain Infrastructures section).
- <u>Violations</u> Law violations cover crimes or infractions at any jurisdiction level (international, national, local) including copyright or intellectual property right infringements. These may lead to criminal cases potentially resulting in jail and/or fines levied, or civil lawsuits potentially resulting in damages to pay.
- <u>People</u> People levels represent pools of individuals performing legal-related functions including their subdivisions such as law creation by elected or appointed officials, legislative staff support, legal enforcement, and judicial personnel; people affected by laws such as overall population levels, victims, incarcerated prisoners, family dependents of incarcerated people, and others.

Level examples may also include quantities such as effort and cost expenditures, fines levied or paid, case schedule dates, personnel attributes such as motivation, staff exhaustion or burnout levels, law amendments and law drafting errors.

There could be many application-specific level types based on the purpose and context of modeled laws. For example, modeling the dynamics of illicit drug laws may entail drug demand levels, the number of cartels, or agricultural resource levels of cartels as demonstrated in (Olaya and Angel, 2014). When the intent of a regulatory law is to prevent bodily injury or harm, then evaluating its effectiveness may necessitate modeling injuries, deaths, hospital stays, health costs incurred, etc.

Sources and sinks represent levels or accumulations outside the boundary of the modeled system. Sources are infinite supplies of entities and sinks are repositories for entities leaving the model boundary. Typical examples for lawmaking sources could be needs for new regulations originating in society or business at-large, or the generation of court filings to be handled. Sinks could represent final judgments of cases leaving court dockets or legal personnel attrition repositories for retirees.

Rates in the lawmaking process are necessarily tied to the levels. Levels don't change without flow rates associated with them. Some examples include law-writing rates, law change rates, case turnover rates, infraction rates, personnel hiring and retiring rates.

Auxiliaries often represent "score-keeping" variables. Examples for tracking purposes include the percent of infractions per population level, percent of injuries or deaths per population, case progress measures, percent of cases in legal states, other ratios or percentages used as independent variables in dynamic relationships.

GENERIC FLOW PROCESSES

Generic flow processes are the smallest, essential structures based on a rate/level system that model common situations and produce characteristic behaviors. They consist of levels, flows, sources/sinks, auxiliaries and sometimes feedback loops.

See the following summaries of generic flows and example applications. Equations are shown for the cases where relations exist with other variables that drive characteristic behavior patterns.

Rate and Level System

The simple rate and level system (also called stock and flow) is the primary structure from which all others are derived. See Figure 2. This system has a single level and a bi-directional flow that can fill or drain the level. Subsequent structures each build on top of this basic structure with additional detail and characteristic behavior.



Flow Chain with Multiple Rates and Levels

The single rate and level system can be expanded into a flow chain incorporating multiple levels and rates. See Figure 3. It can be used to model a process that accumulates at several points instead of one, and is also called a cascaded level system. A generic flow chain within itself does not produce characteristic behavior without other structure and relationships.



Figure 3. Flow Chain with Multiple Rates and Levels

Compounding Process

The compounding structure is a rate and level system with a feedback loop from the level to an input flow, and an auxiliary variable representing the fractional amount of growth per period. See Figure 4. A compounding process produces positive feedback and exponential growth in the level. Modeling applications include the initial rapid escalation of paperwork due to a new ordinance, compounding of new laws to fix previous laws, legal or illicit market dynamics, social communication patterns (e.g. rumors, panic), etc.



Rate = Level * Growth Factor

Figure 4. Compounding Process

Draining Process

Draining can be represented similarly as the compounding process, except the feedback from the level is to an outflow rate and the auxiliary variable indicates how much is drained in the level. See Figure 5. Draining is a common process that underlies delays and exponential decays. Case promotions, fine payments, personnel retirement, skill loss and many other trends can be modeled as draining processes.



Outflow = Level * Draining Fraction

Figure 5. Draining Process

Production Process

A production process represents work accomplished at a rate equal to the number of applied resources multiplied by the resource productivity. See Figure 6. It typically has

an inflow to a level that represents production dependent on resource amounts, which may be a level in an external flow chain representing resources. E.g., the productivity of levying traffic tickets can be modeled this way as a function of police employed.



Production Rate = Resources * Productivity

Figure 6. Production Process

Adjustment Process

An adjustment process is an approach towards goals or equilibrium. The structure contains a goal variable, a rate, level, and adjusting parameter. See Figure 7. The structure models the closing of a gap between the goal and level. The change is more rapid at first and slows down as the gap decreases. The inflow is adjusted to meet the target goal. This basic structure is at the heart of many policies and other behaviors.



Inflow = (Goal - Level) * Adjustment Fraction

Figure 7. Adjustment Process

Co-Flow Process

Co-flows are a shortened name for coincident flows; flows that occur simultaneously through a type of slave relationship. The co-flow process has a flow rate synchronized with another host flow rate, and normally has a conversion parameter between them. See Figure 8. This process can model the co-flows of laws and infractions, laws and associated paperwork, resource tracking such as effort expenditure, or tracking revenues as a function of traffic tickets levied.



Split Flow Process

The split flow process represents a flow being divided into multiple sub flows, or disaggregated streams. It contains an input level, more than one output flow, and typically has another variable to determine the split portions. See Figure 9. Applications include litigation chains to differentiate prosecution case successes vs. failures, other court judgments won vs. lost, or personnel flows to model legal personnel resource allocation to different activities.



Cyclic Loop

A cyclic loop represents entities flowing back through a loop. See Figure 10. The difference from non-closed chains is that a portion of flow goes back into an originating level. This structure is appropriate to represent law amendments, retried cases, habitual re-offenders, and other cycling phenomena.



EXAMPLE GENERIC FLOWS

Figure 11 shows an example of a basic production structure applied to lawmaking. This structure associates multiple personnel resource levels with bill production. It starts with a number of bills to be written, and the bill writing rate uses the number of applied resources (the legislative staff sizes) multiplied by their respective productivities adjusted for experience levels. The staff transition through the experience levels with an average assimilation time and the overall productivity is affected.



Figure 11. Example Legislative Production Structure

The productivity of legislation could be measured with different units. Traditionally it is bills per time unit as this example, but a more normalized "product" could be bill pages to account for different size bills. Example empirical data on the bills/year and bill pages/year for U.S. Congress per (GovTrack, 2016) could be used to calibrate or validate productivity models.

Figure 12 shows an example split flow process for crime detection. This generic flow could be part of a larger model for the laws and enforcement levels that affect crime detection efficiency and the initial commitment rates. The detected crimes could flow into another model portion for the prosecution aspects.

This structure allows policy analysis in terms of setting punishment levels for the deterrence factor. The resources expended on crime detection can also be varied.



Figure 12. Example Crime Detection Model Structure

INFRASTRUCTURES AND BEHAVIORS

The infrastructures in Table 1 are based on one or more of the generic flow types with additional structural details. The additional structure typically leads to characteristic dynamic behaviors. A few of the structures herein do not cause specific dynamic behaviors, but instead are used for intermediate calculations, converters or instrumentation of some kind.

Decision structures are often represented within these structures. These may include policies to allocate legal staff, adjust legal policy goals as enforcement progresses, case turnaround policies, etc. Goals may include desired bill turnaround times, crime or injury reductions, etc.

Table 1. Example Infrastructures and Behaviors withExamples

Infrastructure	Description and Examples
Exponential	Growth structures are based on the
Growth	generic compounding flow process.
	Examples are legal paperwork
	escalation or new crime markets (see
	the compounding process).
S-shaped Growt	An S-shaped growth structure contains
and	at least one level, provisions for a
S-curves	dynamic trend that rises and another
	that falls. There are various
	representations because S-curves may
	result from several types of process
	structures representing the rise and
	fall trends. Examples are cumulative
	cost to establish new laws or
	enforcement knowledge diffusion.
	Deterrence against penalty levels
	exhibits the diminishing returns in S-
	curves.
Delays	Delays are based on the generic
	draining process. A classic example is
	the time delay to try a case. Exponential
	decay results when the outflow
	constant represents a time constant
	from a level that has no inflows. The
	decay declines exponentially towards
	zero. A higher order delay behaves like
	a connected series of first order delays.
Balancing	Balancing feedback (also called
Feedback	negative feedback) occurs when a
	system is trying to attain a goal, such as
	a minimum threshold of injuries via
	regulation or an enforcement hiring
0 111 11	goal.
Oscillation	Oscillating behavior may result when
	there are at least two levels in a system.
	Normally there is a parameter for a
	target goal that the system is trying to
	reach, and the system is unstable as it
	tries to attain the goal. Goals may

	represent desired law effects or resources levels. Examples are oscillating crime rates or personnel systems
Smoothing	An averaging over time. Random spikes will be eliminated when trends are averaged over a sufficient time period. Examples are perceived safety from crime or opportunity for it from the criminal perspective.
Integrated Production Structure	This infrastructure combines elements of the task production and human resources personnel chains. Production is constrained by both productivity and the applied personnel resources external to the product chain. The level of personnel available is multiplied by a productivity rate.
Learning Curve	The continuously varying effect of learning can be modeled via a classic feedback loop between the completed tasks and productivity, to account for becoming more proficient at a legal task. It occurs on individual and organizational levels.
Attribute Tracking	Important attributes to track are frequently calculated from levels. They can be used as inputs to other model portions, such as a decision structure. For example, normalized incarceration rate is calculated by dividing incarcerations by the total population size.
Attribute Averaging	A structure for attribute averaging (similar to attribute tracking) calculates a weighted average of an attribute associated with two or more levels.
Effort Expenditure Instrumentation	Effort or cost expenditures are co-flows that can be used whenever effort or labor cost is a consideration. Frequently this structure serves as instrumentation to obtain cumulative effort and does not play a role in the dynamics of the system. It could be used for decision making in actual processes or measuring cost for comparative purposes.
Decision Structures	Infrastructures for decision policies frequently determine rates. Some common decision structures relevant to lawmaking processes include desired enforcement staff, legal resource allocation, or scheduled case completion date

EXAMPLE INFRASTRUCTURES AND BEHAVIORS

An example structure for a first order delay is shown in Figure 13 that models outflow from a level as introduced in Table 2. It models a batch of bills to process with a time delay. The resulting behavior is in Figure 14. The equation expresses the bill processing outflow rate as a function of the bill level and average legislative delay time. It produces the characteristic exponential decline shown in Figure 14 for a starting level of 10 bills and average delay time of 90 days. This is a simplified example doesn't account for new bills coming in, but the same structure is used when an inflow rate is attached to the initial bill level.



Bill Processing Rate = Bills / Legislative Delay Time





Figure 14. Example First Order Delay Behavior

Figure 15 shows an information smoothing infrastructure modeling perceived crime opportunity as short intermittent interdictions are held. The behavior is in Figure 16. Opportunity is the degree to which criminals feel safe to commit crimes without being caught. When interdiction occurs it takes a delay time to adjust their perception afterwards. The policy implications for lawmaking are the interdiction timing and force levels with limited resources.







Figure 17 shows an example oscillating system demonstrating the cycles of criminals and continuous security forces seen in Figure 18. This example is based on a predator-prey model. The oscillation derives from the cat-and-mouse dynamics between the two levels of continuously embedded security and criminal populations.



Figure 17. Example Oscillating Structure for Criminal and Security Populations



Figure 18. Example Oscillating Behavior

EXAMPLE FLOW CHAIN INFRASTRUCTURES

This section identifies flow chain infrastructures related to lawmaking processes consisting mostly of cascaded levels for legal artifacts, infractions and people. These infrastructures can be used as pieces in a comprehensive lawmaking process model, or could serve as standalone base structures for isolated experimentation.

The chains represent basic flows pervasive in lawmaking processes. When applying system dynamics, the question must be asked: What is flowing? Determination of what kinds of entities flow through a lawmaking process is of primary importance to identify the chains to build models on top of. As always when modeling with system dynamics, the level of aggregation used in the chains depend on the modeling goals and desired level of process visibility.

Laws become transformation sequences modeled as conserved flows, where each level has the same unit, or in non-conserved flow chains where transformation steps are modeled using distinct artifact types for the stages of new legislation. Each level has different units in non-conserved chains. If the lawmaking modeling goals dictate that sequential legislation artifacts be modeled in their respective units then non-conserved flows are used.

Violations include crimes or infringements resulting in jail time, fines or suits to settle. Violation counts are an important law process measure that can provide many insights on law efficiencies and dynamics. There are a number of ways to represent infractions including their generation, detection and case resolutions. Infractions are the primary focus in the chains, but are inextricably tied to other aspects such as law production, enforcement practices, etc.

People flows are conserved flow chains traditionally accounting for sequential experience or promotion pools. Chains for personnel are mainstays of models to account for legal labor and may correspond to attributes for different skillsets, or other differentiators requiring more detail than auxiliaries or single levels can provide. Frequently the chains contain two or more experience levels (e.g. rookies vs. experienced policemen). Varying

degrees of detail and enhancements are possible, such as adding chain splits for attrition from any experience level.

Some introductory examples of flow chains are provided as illustrations. A top-level example of a standard process for lawmaking is in Figure 19. It shows the different stages of laws from enacted to repealed. The flow chain houses many laws moving through their states and they accumulate in the levels. The number of laws in any level could be counted at a given instant of time. Not shown is the external bill introduction process before they are made laws (as in Figure 20 following).

The lawmaking flow chain is a skeleton that could be augmented with additional detail of relationships. For example, the delay structure in Figure 3 for legislative delay could be a component of such a flow chain.

An example of a flow chain for a state legislative process for a house is in Figure 20. This example is modeled after the California processes for the Senate and Assembly (State of California, 2016). It is simplified by not showing the possible iterations of committee amendments. This process would be repeated in both houses and then requires final resolutions of differences and the Governor's signature (not shown). It could be augmented with the actual observed delays, bill introduction rates and other representative rate patterns.

Figure 21 models a common criminal justice process flow. Not shown are possible separate flows for felonies and misdemeanors, and additional activities beyond the boundary for penitentiary and parole states. The rates and levels could be expanded for these additional considerations.

Figure 22 shows a flow chain infrastructure for Intellectual Property Rights (IPRs) modified from (Derwisch and Kopainsky, 2010). It shows the states of IPRs from initial application through court cases for infringement. It is not assumed that all IPRs go through each level, and some may traverse no further than the middle levels.

The added detail on this infrastructure shows some nearest neighbor variables affecting the rates. Not included are the other connections relating the variables. This example illustrates how the basic infrastructures can be incrementally built out with supporting detail.



Figure 19. Lawmaking Flow Chain



Figure 20. State Legislative Process Flow Chain



Figure 21. Criminal Justice Process Flow Chain





CONCLUSIONS AND FUTURE WORK

This work provides reusable model structures interpreted and tailored for the lawmaking process domain. The hierarchy of model structures and patterns provides a taxonomy for lawmaking applications. Characteristic behavior patterns over time are encapsulated with their causal structures.

The reusable model assets have been the result of culling lawmaking and related processes and are convenient for creating new model applications. Lawmaking personnel, electees or officials in legislative and regulatory bodies, legal scholars, public policy researchers, other legal practitioners and students are encouraged to use and experiment with them.

Modelers can save time by leveraging existing and wellknown patterns. The generic structures are starting templates that can be combined in different ways, and with detail added to create larger infrastructures and complex models. The building blocks help lower the barrier of adoption in the community because they can be quickly reused and adapted for numerous applications.

This author will continue improving these modeling assets, developing fuller models for specific investigations and seeking actual data to support the modeling. The generic structures, sample flow chains and models will be provided in the public domain.

Subsequent work will include small scale models demonstrating system archetypes in lawmaking, such as showing how unintended consequences of laws occur. More elaborated, complete model applications will also be provided. Web-based, executable versions will be accessible for public usage of the lawmaking applications.

This author is collaborating with the Science of Laws Institute to provide public models and resources. Readers can check http://www.scienceoflaws.org/models or http://sdsim.com/models/lawmaking and we invite your feedback and suggestions.

Further detailed investigations into IPR laws are underway (adapting Figure 22). This will be reported in subsequent work with a focus on software intellectual property. The modeling has some commonalities with (Derwisch et al., 2010) on IPR dynamics and (Bodner, 2015) for international aspects.

Empirical data collection for developing and validating lawmaking models will also be undertaken. Actual data on all aspects of lawmaking is critical, and must be continuously sought for solid model underpinnings.

Public records can provide much data on legislation and its impacts. For example, data on the U.S. Congress bill passage rate for the last few decades (GovTrack, 2016) provides actual rates of bill passages, bill page sizes, etc. in order to calibrate such models. Crime and incarceration statistics are readily available. But there is also data hidden or held "close to the vest" for some legislative processes where more transparency is needed.

This paper is a beginning as there are numerous law topics to investigate aided by simulation. It is hoped to catalyze interest in the field, and provide guidance on one approach for applying science for better lawmaking.

The application gamut spans local, national and international legislative processes. Thousands of specific laws (current and proposed) warrant detailed study and analysis. High-level models can also be developed for the lawmaking trade space. For example, trying to determine the "sweet spot" of the optimal number of laws as a societal risk balance.

The models are for insight and impact, not just for play. The goal is to interject use of models and simulation into actual legislative practice. Eventually we hope that modeling and simulation of lawmaking will become adopted as an inherent part of the process and standard professional practice.

REFERENCES

- Bodner, Douglas A. "Mitigating Counterfeit Part Intrusions with Enterprise Simulation", Procedia Computer Science 61, Elsevier (2015): 233 – 239
- Derwisch, Sebastian and Kopainsky, Birgit. "Dynamics of Enforcement and Infringement of Intellectual Property Rights and Implications for Innovation Incentives", Proceedings of the 28th International Conference of the System Dynamics Society (2010)
- Forrester, Jay W. *Principles of Systems*. Cambridge, MA: MIT Press, 1968.
- Forrester, J. W., *Urban Dynamics*, Cambridge MA: Productivity Press, 1969
- Ghaffarzadegan, Navi, et al. "How Small System Dynamics Models Can Help the Public Policy Process", System Dynamics Review 27(1): 22-44., System Dynamics Society, 2011.
- GovTrack, "Historical Statistics about Legislation in the U.S. Congress", https://www.govtrack.us/congress/bills/statistics (last accessed October 2016)
- Hines, Jim. *Molecules of Structure Version 1.4*, LeapTec and Ventana Systems, Inc., 2000.
- Levin, G., et al. The Persistent Poppy: A Computer-Aided Search for Heroin Policy. Cambridge MA: Ballinger, 1975.
- Madachy, Raymond J. Software Process Dynamics, Hoboken, NJ: Wiley-IEEE Press, 2008.
- Meadows, Donella H., et al. *The Limits to Growth*, New York: Universe, 1972.
- Meadows, Donella H., et al. *Limits to Growth-The 30 year* Update, White River Junction, VT: Chelsea Green, 2004.
- Morecroft, J. D., "System Dynamics and Microworlds for Policymakers", European Journal of Operational Research 35 (3): 301-320, 1988.
- Olaya, Camilo and Angel, Vanessa. "The War on Drugs: A Failure in (Operational) Thinking", Proceedings of the 32nd International Conference of the System Dynamics Society (2014)
- Richmond, Barry, et al. *Ithink User's Guide and Technical Documentation*, High Performance Systems Inc., Hanover, NH, 1990.

Madachy

- Schrunk, David G. The End of Chaos: Quality Laws and the Ascendancy of Democracy. Poway, CA: QL Press, 2005.
- Senge, Peter. *The Fifth Discipline*. New York, NY: Doubleday, 1990.
- State of California, "OVERVIEW OF LEGISLATIVE PROCESS", http://www.leginfo.ca.gov/bil2lawx.html (last accessed October 2016)
- Sterman, John D. Business Dynamics: Systems Thinking and Modeling for a Complex World, Boston, MA: Irwin/McGraw-Hill, 2000.

APPENDIX A: MATHEMATICAL FORMULATION OF SYSTEM DYNAMICS

The mathematical structure of a system dynamics simulation model is a set of coupled, nonlinear, first-order differential equations,

$$\mathbf{x}'(\mathbf{t}) = \mathbf{f}(\mathbf{x}, \mathbf{p}),$$

where \mathbf{x} is a vector of levels, \mathbf{p} a set of parameters and \mathbf{f} is a nonlinear vector-valued function. As simulation time advances, all rates are evaluated and integrated to compute the current levels.

Runge-Kutta or Euler's numerical integration methods are

$$Level = Level_0 + \int_0^t (inflow - outflow) dt$$

normally used for determining levels at any time *t* based on their inflow and outflow rates:

The dt parameter corresponds to the chosen time increment for execution. Corresponding system dynamics code for the level calculations would be:

Level(time) = *Level*(time-dt) + (*inflow* - *outflow*)*dt

INIT Level=Level₀

where *Level* is computed for any time, *Level*₀ is the initial level value, and the flow rates to/from the level are *inflow* and *outflow* respectively. Describing the system with equations like the above spares the modeler from integration mechanics.

Note that tools relieve the modeler from constructing the equations; rather a diagrammatic representation is drawn and the underlying equations are automatically produced.



Dr. Raymond Madachy is an Associate Professor in the Systems Engineering Department at the Naval Postgraduate School. His research interests include system and software cost modeling, affordability and tradespace analysis, modeling and simulation of systems and software engineering and lawmaking processes, integrating systems engineering and software engineering disciplines, software litigation and intellectual property rights. Previously he was a Research Assistant Professor in the Industrial and Systems Department at the University of Southern California and a Principal in the USC Center for Systems and Software Engineering. He has over 90 publications including the book *Software Process Dynamics*, is a co-author of *Software Cost Estimation with COCOMO II* and *Software Cost Estimation Metrics Manual for Defense Systems*. He is currently writing *Systems Engineering Principles for Software Engineers*.