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Empirical Cost Estimation Tool (PMS 320)

17 October 2016

Dr. Johnathan Mun, Professor of Research, Information Science

Dr. Thomas Housel, Professor, Information Science

Graduate School of Business & Public Policy

Naval Postgraduate School

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Prepared for the Naval Postgraduate School, Monterey, CA 93943.



ACQUISITION RESEARCH PROGRAM
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Executive Summary

This research project pertains to the development of alternative ship cost modeling methodologies. Most ship cost modeling has been traditionally weight-based. This approach drives the U.S. Navy to select smaller ships that, consequently, require custom-designed shipboard components. This research project is intended to help determine if there is a more accurate way to empirically predict, forecast, and model ship cost. Current and forecasted Department of Defense (DoD) budgets require identifying, modeling, and estimating the costs of shipbuilding. Information and data were obtained via publicly available sources, and were collected, collated, and used in an integrated risk-based cost and schedule modeling methodology. The objective of this study is to develop a comprehensive cost modeling strategy and approach, and as such, notional data were used. Specifically, we used the Arleigh Burke Class Guided Missile Destroyer DDG 51 Flight I, Flight II, Flight IIA, and Flight III as a basis for the cost and schedule assumptions, but the modeling approach is extensible to any and all other ships within the U.S. Navy. The results will be used to develop recommendations and develop a cost modeling tool on how to implement ship cost forecasts. This example will provide a roadmap for other new ship cost modeling by the U.S. Navy, thereby improving effectiveness and increasing cost savings.



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Introduction

This research project pertains to the identification, review, and potential development of existing and alternative ship cost modeling methodologies. Most ship cost modeling has been traditionally weight-based. This approach drives the U.S. Navy to select smaller ships that, consequently, require custom-designed shipboard components. This research project is intended to help determine if there is a more accurate way to empirically predict, forecast, and model ship cost. Current and forecasted Department of Defense (DoD) budgets require identifying, modeling, and estimating the costs of shipbuilding. The results will be used to develop recommendations and develop a cost modeling tool on how to implement ship cost forecasts. This example will provide a roadmap for other new ship cost modeling by the U.S. Navy, thereby improving effectiveness and increasing cost savings.

The main crux of this research project is that the researchers will review and present some of the most applicable cost modeling methodologies, and generate some notional cost data (rough order magnitude values will be collected or generated). These “data” will be generated by the researchers using previous actual cost data from ship maintenance projects of various DDGs. We will make sure the report clearly states this extrapolation, and we will apply said generated data to the various cost models. This way, readers and relevant sponsors can see the various types of cost models, approaches, and sample data variables that are required to run said models, sample results, as well as the pros and cons of each approach. We can always pursue a follow-on project in the subsequent year if there is a method that is of interest or that the sponsors feel might be applicable. The required data variables as well as sample results will be listed in the report, so the sponsors will know what to expect ahead of time. We can then obtain real-life cost data to plug into the models. The cost model approaches may include the standard parametric models, nonparametric methods, systems dynamics based on project management task-based schedule and cost models, semiparametric Monte Carlo simulation models, curve fitting, time-series and cross sectional models, and nonlinear models, and so forth.



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Research Steps

The proposed research will use the following steps.

Task 1: Literature Review

Mun and Housel, with assistance from Naval Postgraduate School (NPS) students and research associates, will review the existing literature in terms of ship development costing. First, literature on existing shipbuilding processes will be collected, reviewed, and used to develop a comprehensive shipbuilding process that is generic and applicable in general for the U.S. Navy, while at the same time modular in nature to handle customizable and module ship development. Second, literature on existing cost estimation methods for shipbuilding and other related large-scale manufacturing processes will be collected, reviewed, and used to develop a list of potential critical success factors and cost estimation methods that can be considered by the U.S. Navy.

Task 2: Implementation Assessment Framework

Mun, Housel, students, and research assistants will review the potential barriers, past approaches, and critical success factors developed in Task 1, which will eventually be used to develop an implementation assessment framework. This framework will structure the barriers and critical success factors in a way that relates those concerning similar aspects of implementation (e.g., to implementation activities or technology characteristics) and facilitates data collection, data analysis, and recommendations.

Task 3: Data Collection or Recommendation

Mun, assisted by Housel and graduate students, will use the implementation assessment framework as the basis for data collection on new, current, and past ship development and maintenance cost. Data will be collected from practitioners about two issues: previous cost estimation models and the actual empirical costs of ship development or ship maintenance costs. Data will be collected primarily from U.S. Navy personnel and ship personnel who are knowledgeable about cost



estimation and actual costing of new ships, as well as from existing secondary research. In the event that relevant data are not readily available, the researchers will be using existing ship maintenance data and extrapolate them to resemble ship build costs (rough order estimates will be used and the assumptions used to extrapolate and generate said data will be clearly enumerated in the report). Data from relevant industry applications will be collected if and as it is available to augment U.S. Navy data. Data on ship maintenance will also be collected.

Task 4: Cost Model Creation

Mun and Housel will use the collected data or newly generated data for analysis to develop an improved understanding of the barriers to and critical success factors for a better and more accurate cost estimation model. This analysis will include organizing the data into the implementation assessment framework and the comparison of data across sources using descriptive statistics, econometric modeling, and analytical costing models.

Task 5: Development of Costing Tool

Examples of cost estimation models using existing tools such as Risk Simulator will be developed that will incorporate stochastic predictive modeling using historical data or sample generated data to back-fit forecast models to generate future forecasts coupled with Monte Carlo risk simulation methods to generate probability distributions of the cost estimates. This task represents the bulk of the work in this research project.

Task 6: Model Development and Recommendations for Implementation

Mun and Housel will generate and review the results of the implementation assessment from Tasks 4 and 5 to make recommendations to the U.S. Navy about how to best implement the new cost estimation model. Those recommendations may include specific barriers encountered in the study, critical success factors to create or reinforce, and potential strategies for implementing the new cost estimation approach. The revised cost estimation model and approach will be ready for large-



scale implementation. The primary product of this task will be written recommendations for implementation.

Task 7: Prepare Reports and Presentations

Mun, Ford, and Housel will prepare a report and presentation of the key results and conclusions for wide distribution. The report and presentation will also be suitable for submission to the Acquisition Research Symposium in 2017.



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Literature Survey

In the *NAVSEA Cost Estimation Handbook*, the author provides a ready reference to “support the stewardship of our cost engineering capabilities” (Deegan, 2005), while Independent Cost Estimating Services by SPAR Associates (2015) “uses its system to quickly estimate ship costs based on initial design data and to provide the impact on costs of alternate design and build strategy decisions.”

Lee (2014) looked at improving the parametric method of cost estimating relationships of U.S. Navy ships. In considering recent military budget cuts, there has been a focus on determining methods to reduce the cost of U.S. Navy ships. In accordance to Lee,

RAND National Defense Research Institute study showed many sources of cost escalation for Navy ships. Among them included characteristic complexity of modern Naval ships, which contributed to half of customer driven factors. This paper focuses on improving the current parametric cost estimating method used as referenced in NAVSEA’s Cost Estimating Handbook.

Currently, as Lee (2014) describes,

Weight is used as the most common variable for determining cost in the parametric method because it’s a consistent physical property and most readily available. Optimizing ship design based on weight may increase density and complexity because ship size is minimized.

That paper introduced “electric power density and outfit density as additional variables to the parametric cost estimating equation and will show how this can improve the early stage cost estimating relationships of Navy ships” (Lee, 2014).

From our literature survey, we found that there are four common types of cost estimating methods: “Analogy, Parametric, Engineering Build-up, and Extrapolation from Actuals” (Lee, 2014). During the very early stages of cost estimating, even before the concept refinement stage, the Analogy cost estimating method is used.



As more details emerge and more information is available for the cost estimator, a more accurate, Build-up cost estimation is used. Toward the end of the ship's lifecycle, we can Extrapolate actual cost information, and it is no longer an estimation.

NAVSEA (Naval Sea Systems Command [NAVSEA], 2015) released instructions regarding the preparation of government cost estimates. The general methods described in the manual include the four most common methods of cost estimating: "roundtable, comparison, detailed estimating, and parametric cost estimating (cost estimate relationships)."

In his article "Budget Office Questions Navy Shipbuilding Cost Estimates," Walcott (2012) finds that the U.S. Navy is

underestimating the cost of its proposed 30-year shipbuilding program by 19 percent, the non-partisan Congressional Budget Office said in a report. By comparison, using its own models and assumptions, CBO estimates that the cost for new-ship construction under the 2013 plan would average \$20.0 billion per year, or a total of \$599 billion through 2042.

In *Integrating Cost Estimating with the Ship Design Process*, Deschamps and Greenwell (2009) explain that the ship design process is an

evolutionary process where at the conceptual design level, pre Milestone A for Naval acquisition programs, few details are known and the metrics used for estimating costs are based on analogous platforms and limited parametric functions. As the design process continues towards Milestone B the design begins to take shape with fewer analogies and an increasing number of parametric cost drivers. At this point, 80% of the life cycle costs (LCC) are set and the cost risk associated with the design becomes an important piece of the overall acquisition costs. It is imperative that the methods used to estimate the cost and cost risk are tightly coupled with the design iteration process and are parametric in nature in order to support the needs of the Program Manager in terms of not only the basic design but design trade-offs.



The authors present the use and benefits of employing a set of parametric cost models during the concept and preliminary phases of ship design.

These cost models produce quick assessments of costs and risk, for design and mission trade-off alternatives. The cost models, being parametric, can follow the evolutionary design process. At early stages of the design, when many details of the design are not yet available, the cost models automatically provide statistically-synthesized values for missing parameters. Then, as the design matures, these default values can be replaced with values developed for the design. (Deschamps & Greenwell, 2009)

In *A Practical Approach for Ship Construction Cost Estimating*, Ross (2002) states that to succeed commercially, shipyards must be able to accurately estimate costs. Cost estimating is necessary for the “bid process, change orders, and trade-off studies.” Numerous cost estimating approaches exist. They are based on extrapolations from “previously-built ships, detailed bottoms-up parametric models, and integrated physics-based analyses.” Cost estimating can be frustrating to shipyard personnel. Cost estimators may lack timely technical information and face data inconsistencies.

Ship engineers and naval architects commonly lack feedback on the cost consequences of their technical decisions. Managers often lack information denoting the level of confidence in cost estimates upon which they must make business decisions. Finally, many approaches to cost estimating are mysterious and not formally validated (each cost estimator has his own black book), complicated (too time consuming to be of use to decision makers), or difficult to use (steep learning curve). (Ross, 2002)

This paper presents an approach that enables instant sharing of cost and technical data among ship engineers, naval architects, and cost estimators; the analysis was meant to provide confidence measures to managers.

Truver (2001) believes that estimating ship construction costs is behind the times. In one highly critical area of naval analysis, the U.S. Navy seems to be “bogged down in the early years of the last century.” The U.S. Navy’s traditional



approach and methodology for estimating the construction and lifecycle costs of new ships is “out of step with the Revolution in Business Affairs.” According to Truver (2001):

The Naval Surface Warfare Center (NSWC) is rethinking the current paradigm of ship cost estimating. Taking the lead in a joint Navy-industry initiative to reinvent the way ship costs are determined, have developed the Product Oriented Design and Construction (PODAC) Cost Model.

Since the end of the Cold War, naval procurement for the U.S. Navy has seen a dramatic decrease. This decrease in defense spending has placed existing programs under more scrutiny than previous years. As a result, there is less tolerance on the part of taxpayers and U.S. Congress for procurement cost growth. (Miroyannis, 2006)

The research attempts to examine the current method that the U.S. Navy conducts ship cost estimates, and it suggests changes in order to improve the confidence level and accuracy of the forecasts. An examination of how industry is conducting cost estimates was used as a comparison to the current U.S. Navy practices. Finally,

using only a weight based approach to ship cost estimating is insufficient. It is necessary to develop and use a model that incorporates other cost driving factors in order to develop estimates of sufficient quality at the preliminary design level. (Miroyannis, 2006)

Smith (2008) updates one ship cost estimation model by combining the two existing models (the Basic Military Training School [BMTS] Cost Model and the MIT Math Model) in order to develop a program that can accurately determine both a ship's acquisition cost as well as its life cycle cost. Using United States Coast Guard resources, this project addressed various aspects of the ship design process which have a direct effect on the cost of building a ship. This will include, but not be limited to, the cost estimation process, determining which design decisions have the biggest impact on the ship's total cost, common pitfalls in the design process that lead



to increases in cost, and lessons learned that have helped minimize the cost of a ship.

Sullivan (2011) found that the

inability to predict ship acquisition cost accurately is a great impediment to budget formulation and execution for shipbuilding programs. It also has eroded the U.S. Navy's credibility with Congress. Dramatic improvements in cost analysis tools are needed. Areas for improvement include the following:

- Prediction of R&D costs based on system complexity, subsystem technology, and state of development;
- Modeling of design and construction workforce requirements;
- 10 Naval Ship Design and Construction;
- Topics for the Research and Development Community;
- Modeling the cost of design tools, including configuration, mass properties tools;
- Product Logistics Models environment;
- Modeling of ship integration and test costs;
- Assessment of the costs of facilitation of prime shipbuilding contractor, principal subcontractors, and warfare system contractors;
- Modeling of the effects of concurrent workloads from multiple contracts at all contractors facilities;
- Assessment of cost of government warfare center participation in development and execution; and
- Probabilistic cost analysis tools that give the range of estimates and the probability that the estimates will not be exceeded. (Sullivan, 2011)

Cost estimating tools could benefit from an approach that takes advantage of the massive computing power available today, and also the availability of highly



intelligent search engines. The principle should be that if cost data exist anywhere, the U.S. Navy should be able to access them. This means that the cost of any component or commodity could theoretically be queried, stored in the navy-shipbuilder cost database, and periodically updated, either from catalog information, bid pricing, or other publically available information. The U.S. Navy should, according to Sullivan,

adapt one or more of the commercially available search engines for this purpose and mandate its use for all shipbuilding programs. Furthermore, if shipbuilders could continue to execute the Common Parts Catalog initiative of the National Shipbuilding Research Program (NSRP), the search engines could query this catalog for component cost tabulation. (Sullivan, 2011)

Moore and White (2005) used a regression approach for estimating procurement costs:

Cost growth in Department of Defense weapons system continues to be a scrutinized area of concern. One way to minimize unexpected cost growth is to derive better and more realistic cost estimates. In this vein, cost estimators have many analytical tools to ply. Previous research has demonstrated the use of a two-step logistic and multiple regression methodology to aid in this endeavor. We investigate and expand this methodology to cost growth in procurement dollar accounts for the Engineering and Manufacturing Development phase of DOD acquisition. We develop and present two salient statistical models for cost estimators to at least consider if not use in mitigating cost growth for existing and future government acquisition programs.

According to Brown and Neu (2008), engineering cost models must be reliable, practical and sensitive to the cost and performance impact of producibility enhancements. A baseline surface combatant cost model was developed using a modified weight-based approach. A more flexible model will be developed in Phase 2 using ACEIT (Automated Cost Estimating Integrated Tools). ACEIT is an automated architecture and framework for cost



estimating. It is a government-developed tool that has been used to standardize and simplify the Life Cycle Cost estimating process in the government environment. Core features include a database to store technical and (normalized) cost data, a statistical package specifically tailored to facilitate cost estimating relationship (CER) development and a spreadsheet that promotes structured, systematic model development, and built-in government-approved inflation, learning, time phasing, documentation, sensitivity/what-if, risk and other analysis capabilities. Our task will be to adapt this general framework for concept development naval ship cost analysis including producibility. Cost uncertainty aspects will be integrated with Task 2.3.

The *Joint Agency Cost Schedule Risk and Uncertainty Handbook* (Cost Assessment Data Enterprise [CADE], 2014) states that the government cost analysis community recognizes the need to

capture the inherent uncertainty of acquisition programs into realistic cost estimates to support milestone decision process. Programmatic, cost, schedule, and technical uncertainties are present from the earliest concept exploration phase, through system development, acquisition, deployment, to operational and sustainment. Many estimating processes have focused on producing a single, discrete dollar value that in turn becomes the budget. Realistically, estimating processes develop a range of likely values, with objective and quantifiable analysis of uncertainty intrinsically embedded. The goal of this handbook is to introduce industry best practices for incorporating uncertainty into our estimates in order to provide decision makers with the information necessary to make sound, defensible investment decision.

This handbook emphasizes the need to shift away from estimates based solely on the best-guess of system and programmatic parameters and encourages the cost analyst to build models that address technical, programmatic, cost and schedule uncertainties, and risks as interdependent, not-separate, processes. The effective incorporation of risk uncertainty in cost and schedule estimates is a



challenging task. This handbook is promulgated to help establish a systematic, structured, repeatable and defensible process for delivering comprehensive estimates to Government leadership to get the best possible capability with increasingly limited available resources. (CADE, 2014)

Cost estimating in the Naval Sea Systems Command “requires accurate costs estimates as it is critical to achieving an affordable U.S. Navy shipbuilding program” (Deegan and Mondal, 2008).

There is significant concern, both within and outside the Department of Defense, over the future affordability of the U.S. Navy’s shipbuilding programs. The increasing costs of these programs reflect a variety of factors, such as lower production quantities, increasing weapons system complexity, increasing commodity prices, and a shortage of skilled, workers in the shipbuilding industry. This article examines the challenges one faces when attempting to accurately predict future ship and weapons system costs. It also summarizes current initiatives under way within the cost engineering organization of the Naval Sea Systems Command (NAVSEA) to mitigate these challenges. Reliable cost estimates are important to maintaining a viable Navy. It is encouraging to see greater importance accorded to independent cost estimating within the DON along with efforts to understand and use quantitative risk analysis in making cost decisions. NAVSEA cost estimators are proud to be leaders in this endeavor. (Deegan & Mondal, 2008)

Mulligan (2008) states that

the accepted method for estimating ship construction and operating costs is due to Harry Benford, a professor of naval architecture and marine engineering at the University of Michigan, and dates from the 1960s. Benford conducted regression studies with a variety of technical and cost parameters to arrive at basic algebraic relationships among cargo capacity, ship dimensions, degree of streamlining (block coefficient), design operating speed, Admiralty coefficient, required shaft horsepower, required engine size,



and ship steel weight. His approach however is based on design assumptions which have grown increasingly less applicable.

This research presents new

models for estimating newbuilding costs, based on recent 2003–2007 data. This dataset reflects contemporary ship design and construction practices, and recent cost trends. The models can be used as a basis for economic analysis whenever newbuilding ship cost is considered as an alternative. Though not making an abrupt break with accepted practice, the cost equations presented above offer various advantages for shipping economists and strategic planners. Estimating newbuilding costs with these models captures recent practical experience and cost trends facing the industry in the past few years. (Mulligan, 2008)

Applications of known data mining algorithms to the problem of estimating the cost of ship development and construction was conducted. According to Kaluzny et al. (2011), the work is a product of

North Atlantic Treaty Organization Research and Technology Organization Systems Analysis and Studies 076 Task Group. In a blind, ex post exercise, the Task Group set out to estimate the cost of a class of Netherlands' amphibious assault ships, and then compare the estimates to the actual costs (the Netherlands Royal Navy withheld the actual ship costs until the exercise was completed). Two cost estimating approaches were taken: parametric analysis and costing by analogy. For the parametric approach, the M5 system (a combination of decision trees and linear regression models) of Quinlan (1992). Agglomerative hierarchical cluster analysis and non-linear optimization was used for a cost estimation by analogy approach void of subjectivity.



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The Theory of Predictive Modeling in Cost

Different Types of Forecasting Techniques

Generally, forecasting can be divided into quantitative and qualitative approaches (Figure 1). Qualitative forecasting is used when little to no reliable historical, contemporaneous, or comparable data exist. Several qualitative methods exist such as the Delphi or expert opinion approach (a consensus-building forecast by field experts, marketing experts, or internal staff members), management assumptions (target growth rates set by senior management), as well as market research or external data or polling and surveys (data obtained through third-party sources, industry and sector indexes, or active market research). These estimates can be either single-point estimates (an average consensus) or a set of prediction values (a distribution of predictions). The latter can be entered into Risk Simulator as a custom distribution and the resulting predictions can be simulated; that is, running a nonparametric simulation using the prediction data points as the custom distribution.

For quantitative forecasting, the available data or data that need to be forecasted can be divided into time-series (values that have a time element to them, such as revenues at different years, inflation rates, interest rates, market share, failure rates, and so forth), cross-sectional (values that are time-independent, such as the grade point average of sophomore students across the nation in a particular year, given each student's levels of SAT scores, IQ, and number of alcoholic beverages consumed per week), or mixed panel (mixture between time-series and panel data; e.g., predicting sales over the next 10 years given budgeted marketing expenses and market share projections, which means that the sales data are time-series but exogenous variables such as marketing expenses and market share exist to help to model the forecast predictions).

Here is a quick review of each methodology and several getting started examples in using the software.



- *ARIMA*. Autoregressive integrated moving average (ARIMA, also known as Box–Jenkins ARIMA) is an advanced econometric modeling technique. ARIMA looks at historical time-series data and performs back-fitting optimization routines to account for historical autocorrelation (the relationship of a variable's values over time, that is, how a variable's data is related to itself over time). It accounts for the stability of the data to correct for the nonstationary characteristics of the data, and it learns over time by correcting its forecasting errors. Think of ARIMA as an advanced multiple regression model on steroids, where time-series variables are modeled and predicted using its historical data as well as other time-series explanatory variables. Advanced knowledge in econometrics is typically required to build good predictive models using this approach. Suitable for time-series and mixed-panel data (not applicable for cross-sectional data).
- *Auto-ARIMA*. The Auto-ARIMA module automates some of the traditional ARIMA modeling by automatically testing multiple permutations of model specifications and returns the best-fitting model. Running the Auto-ARIMA module is similar to running regular ARIMA forecasts. The differences being that the required P, D, Q inputs in ARIMA are no longer required and that different combinations of these inputs are automatically run and compared. Suitable for time-series and mixed-panel data (not applicable for cross-sectional data).
- *Basic Econometrics*. Econometrics refers to a branch of business analytics, modeling, and forecasting techniques for modeling the behavior or forecasting certain business, economic, finance, physics, manufacturing, operations, and any other variables. Running Basic Econometrics models is similar to regular regression analysis except that the dependent and independent variables are allowed to be modified before a regression is run. Suitable for all types of data.
- *Basic Auto Econometrics*. This methodology is similar to basic econometrics, but thousands of linear, nonlinear, interacting, lagged, and



mixed variables are automatically run on your data to determine the best-fitting econometric model that describes the behavior of the dependent variable. It is useful for modeling the effects of the variables and for forecasting future outcomes, while not requiring the analyst to be an expert econometrician. Suitable for all types of data.

- *Combinatorial Fuzzy Logic*. Fuzzy sets deal with approximate rather than accurate binary logic. Fuzzy values are between 0 and 1. This weighting schema is used in a combinatorial method to generate the optimized time-series forecasts. Suitable for time-series only.
- *Custom Distributions*. Using Risk Simulator, expert opinions can be collected and a customized distribution can be generated. This forecasting technique comes in handy when the dataset is small, when the Delphi method is used, or the goodness-of-fit is bad when applied to a distributional fitting routine. Suitable for all types of data.
- *GARCH*. The generalized autoregressive conditional heteroskedasticity (GARCH) model is used to model historical and forecast future volatility levels of a marketable security (e.g., stock prices, commodity prices, oil prices, etc.). The dataset has to be a time series of raw price levels. GARCH will first convert the prices into relative returns and then run an internal optimization to fit the historical data to a mean-reverting volatility term structure, while assuming that the volatility is heteroskedastic in nature (changes over time according to some econometric characteristics). Several variations of this methodology are available in Risk Simulator, including EGARCH, EGARCH-T, GARCH-M, GJR-GARCH, GJR-GARCH-T, IGARCH, and T-GARCH. Suitable for time-series data only.



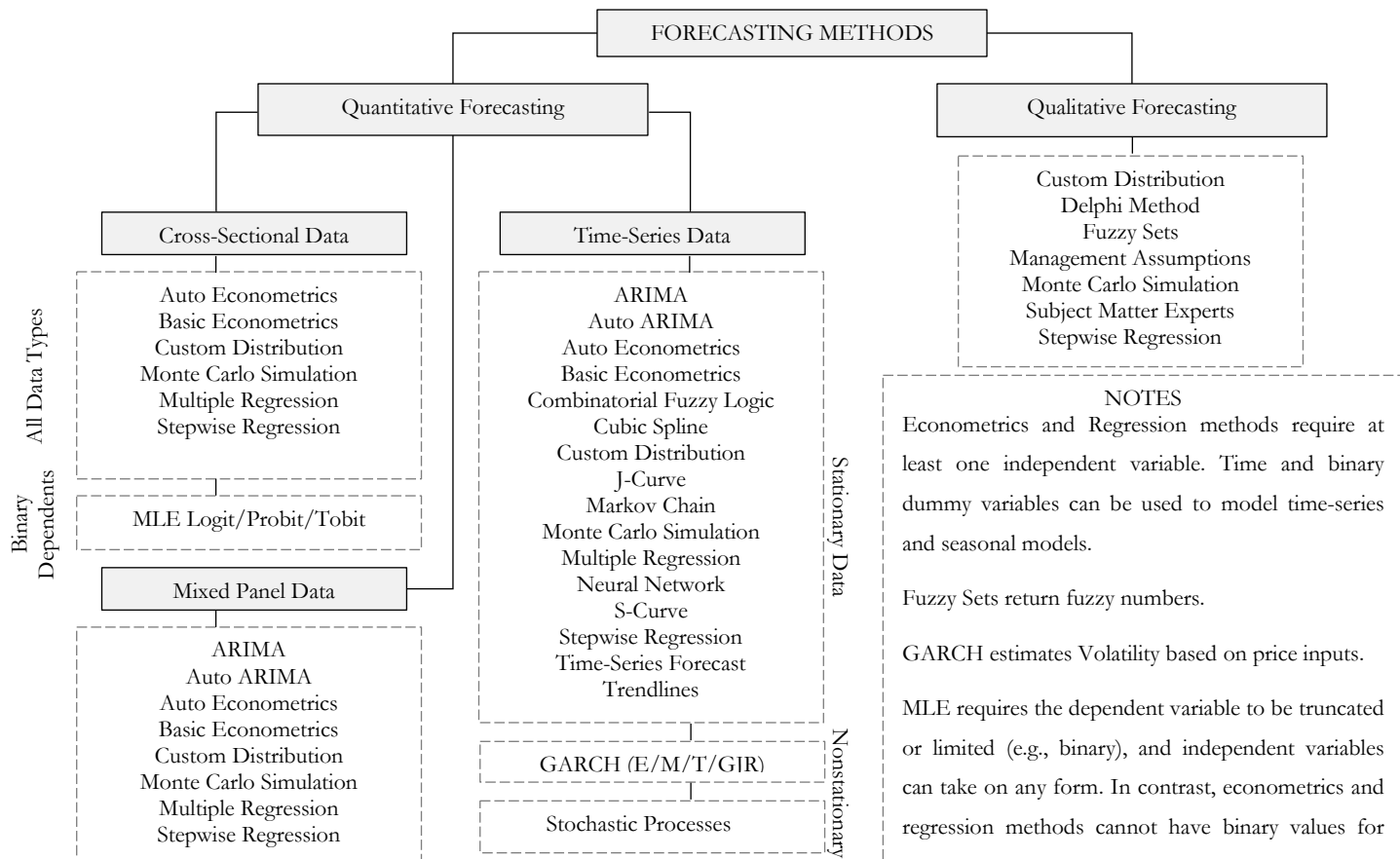


Figure 1: Forecasting Methods



- *J-Curve*. The J-curve, or exponential growth curve, is one where the growth of the next period depends on the current period's level and the increase is exponential. This phenomenon means that over time, the values will increase significantly, from one period to another. This model is typically used in forecasting biological growth and chemical reactions over time. Suitable for time-series data only.
- *Markov Chains*. A Markov chain exists when the probability of a future state depends on a previous state and when linked together forms a chain that reverts to a long-run steady state level. This approach is typically used to forecast the market share of two competitors. The required inputs are the starting probability of a customer in the first store (the first state) returning to the same store in the next period, versus the probability of switching to a competitor's store in the next state. Suitable for time-series data only.
- *Maximum Likelihood on Logit, Probit, and Tobit*. Maximum likelihood estimation (MLE) is used to forecast the probability of something occurring given some independent variables. For instance, MLE is used to predict if a credit line or debt will default given the obligor's characteristics (30 years old, single, salary of \$100,000 per year, and total credit card debt of \$10,000), or the probability a patient will have lung cancer if the person is a male between the ages of 50 and 60, smokes five packs of cigarettes per month or year, and so forth. In these circumstances, the dependent variable is limited (i.e., limited to being binary 1 and 0 for default/die and no default/live, or limited to integer values such as 1, 2, 3, etc.) and the desired outcome of the model is to predict the probability of an event occurring. Traditional regression analysis will not work in these situations (the predicted probability is usually less than zero or greater than one, and many of the required regression assumptions are violated, such as independence and normality of the errors, and the errors will be fairly large). Suitable for cross-sectional data only.



- *Multivariate Regression.* Multivariate regression is used to model the relationship structure and characteristics of a certain dependent variable as it depends on other independent exogenous variables. Using the modeled relationship, we can forecast the future values of the dependent variable. The accuracy and goodness-of-fit for this model can also be determined. Linear and nonlinear models can be fitted in the multiple regression analysis. Suitable for all types of data.
- *Neural Network.* This method creates artificial neural networks, nodes, and neurons inside software algorithms for the purposes of forecasting time-series variables using pattern recognition. Suitable for time-series data only.
- *Nonlinear Extrapolation.* In this methodology, the underlying structure of the data to be forecasted is assumed to be nonlinear over time. For instance, a dataset such as 1, 4, 9, 16, 25 is considered to be nonlinear (these data points are from a squared function). Suitable for time-series data only.
- *S-Curves.* The S-curve, or logistic growth curve, starts off like a J-curve, with exponential growth rates. Over time, the environment becomes saturated (e.g., market saturation, competition, overcrowding), the growth slows, and the forecast value eventually ends up at a saturation or maximum level. The S-curve model is typically used in forecasting market share or sales growth of a new product from market introduction until maturity and decline, population dynamics, and other naturally occurring phenomenon. Suitable for time-series data only.
- *Spline Curves.* Sometimes there are missing values in a time-series dataset. For instance, interest rates for years 1 to 3 may exist, followed by years 5 to 8, and then year 10. Spline curves can be used to interpolate the missing years' interest rate values based on the data that exist. Spline curves can also be used to forecast or extrapolate values of future time



periods beyond the time period of available data. The data can be linear or nonlinear. Suitable for time-series data only.

- *Stochastic Process Forecasting.* Sometimes variables are stochastic and cannot be readily predicted using traditional means. Nonetheless, most financial, economic, and naturally occurring phenomena (e.g., motion of molecules through the air) follow a known mathematical law or relationship. Although the resulting values are uncertain, the underlying mathematical structure is known and can be simulated using Monte Carlo risk simulation. The processes supported in Risk Simulator include Brownian motion random walk, mean-reversion, jump-diffusion, and mixed processes, useful for forecasting nonstationary time-series variables. Suitable for time-series data only.
- *Time-Series Analysis and Decomposition.* In well-behaved time-series data (typical examples include sales revenues and cost structures of large corporations), the values tend to have up to three elements: a base value, trend, and seasonality. Time-series analysis uses these historical data and decomposes them into these three elements, and recomposes them into future forecasts. In other words, this forecasting method, like some of the others described, first performs a back-fitting (backcast) of historical data before it provides estimates of future values (forecasts). Suitable for time-series data only.
- *Trendlines.* This method fits various curves such as linear, nonlinear, moving average, exponential, logarithmic, polynomial, and power functions on existing historical data. Suitable for time-series data only.



Parametric Cost Model Approach

It is assumed that the user is sufficiently knowledgeable about the fundamentals of regression analysis. The general bivariate linear regression equation takes the form of $Y = \beta_0 + \beta_1 X + \varepsilon$, where β_0 is the intercept, β_1 is the slope, and ε is the error term. It is bivariate as there are only two variables, a Y or dependent variable, and an X or independent variable, where X is also known as the regressor (sometimes a bivariate regression is also known as a univariate regression as there is only a single independent variable X). The dependent variable is named as such as it *depends* on the independent variable, for example, sales revenue depends on the amount of marketing costs expended on a product's advertising and promotion, making the dependent variable sales and the independent variable marketing costs. An example of a bivariate regression is seen as simply inserting the best-fitting line through a set of data points in a two-dimensional plane, as seen on the left panel in Figure 2. In other cases, a multivariate regression can be performed, where there are multiple or k number of independent X variables or regressors, where the general regression equation will now take the form of $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \dots + \beta_k X_k + \varepsilon$. In this case, the best-fitting line will be within a $k + 1$ dimensional plane.

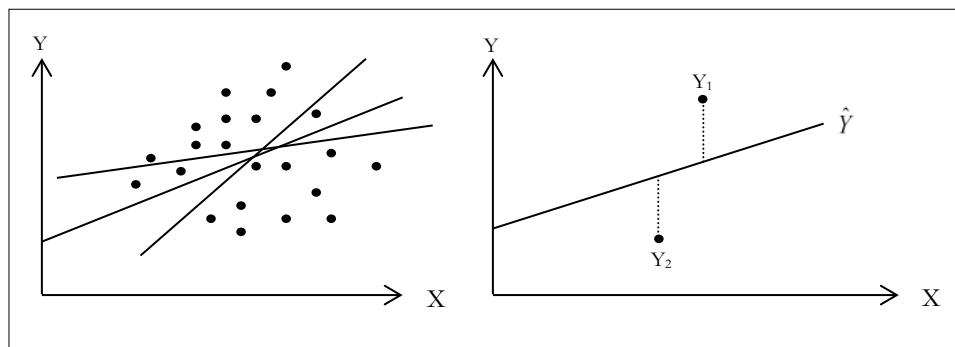


Figure 2: Bivariate Regression

However, fitting a line through a set of data points in a scatter plot, as in the left panel of Figure 8.6, may result in numerous possible lines. The best-fitting line is defined as the single unique line that minimizes the total vertical errors, that is, the sum of the absolute distances between the actual data points (Y_i) and the estimated line (\hat{Y}), as shown on the right panel of Figure 2. To find the best-fitting unique line that minimizes the errors, a more sophisticated approach is applied using regression analysis. Regression analysis finds the unique best-fitting line by requiring that the total errors be minimized, or by calculating

$$\text{Min} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

where only one unique line minimizes this sum of squared errors. The errors (vertical distances between the actual data and the predicted line) are squared to avoid the negative errors from canceling out the positive errors. Solving this minimization problem with respect to the slope and intercept requires calculating first derivatives and setting them equal to zero:

$$\frac{d}{d\beta_0} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = 0 \quad \text{and} \quad \frac{d}{d\beta_1} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = 0$$

which yields the bivariate regression's least squares equations:

$$\beta_1 = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2} = \frac{\sum_{i=1}^n X_i Y_i - \frac{\sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n}}{\sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n}}$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}$$



For multivariate regression, the analogy is expanded to account for multiple independent variables, where $Y_i = \beta_1 + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \varepsilon_i$ and the estimated slopes can be calculated by

$$\hat{\beta}_2 = \frac{\sum Y_i X_{2,i} \sum X_{3,i}^2 - \sum Y_i X_{3,i} \sum X_{2,i} X_{3,i}}{\sum X_{2,i}^2 \sum X_{3,i}^2 - (\sum X_{2,i} X_{3,i})^2}$$

$$\hat{\beta}_3 = \frac{\sum Y_i X_{3,i} \sum X_{2,i}^2 - \sum Y_i X_{2,i} \sum X_{2,i} X_{3,i}}{\sum X_{2,i}^2 \sum X_{3,i}^2 - (\sum X_{2,i} X_{3,i})^2}$$

In running multivariate regressions, great care must be taken to set up and interpret the results. For instance, a good understanding of econometric modeling is required (e.g., identifying regression pitfalls such as structural breaks, multicollinearity, heteroskedasticity, autocorrelation, specification tests, nonlinearities, and so forth) before a proper model can be constructed.

Potential Issues in Parametric Models

The following six assumptions are the requirements for a parametric multiple regression analysis to work:

- The relationship between the dependent and independent variables is linear.
- The expected value of the errors or residuals is zero.
- The errors are independently and normally distributed.
- The variance of the errors is constant or homoskedastic and not varying over time.
- The errors are independent and uncorrelated with the explanatory variables.
- The independent variables are uncorrelated to each other meaning that no multicollinearity exists.

One very simple method to verify some of these assumptions is to use a scatter plot. This approach is simple to use in a bivariate regression scenario. If the assumption of the linear model is valid, the plot of the observed dependent variable



values against the independent variable values should suggest a linear band across the graph with no obvious departures from linearity. Outliers may appear as anomalous points in the graph, often in the upper right-hand or lower left-hand corner of the graph. However, a point may be an outlier in either an independent or dependent variable without necessarily being far from the general trend of the data.

If the linear model is not correct, the shape of the general trend of the X-Y plot may suggest the appropriate function to fit (e.g., a polynomial, exponential, or logistic function). Alternatively, the plot may suggest a reasonable transformation to apply. For example, if the X-Y plot arcs from lower left to upper right so that data points either very low or very high in the independent variable lie below the straight line suggested by the data, while the middle data points of the independent variable lie on or above that straight line, taking square roots or logarithms of the independent variable values may promote linearity.

If the assumption of equal variances or homoskedasticity for the dependent variable is correct, the plot of the observed dependent variable values against the independent variable should suggest a band across the graph with roughly equal vertical width for all values of the independent variable. That is, the shape of the graph should suggest a tilted cigar and not a wedge or a megaphone.

A fan pattern, like the profile of a megaphone, with a noticeable flare either to the right or to the left in the scatter plot, suggests that the variance in the values increases in the direction where the fan pattern widens (usually as the sample mean increases), and this in turn suggests that a transformation of the dependent variable values may be needed.

Dynamic Project Management Approach to Measure and Model Cost-Schedule Risk

In the world of project management, there are essentially two major sources of risks: schedule risk and cost risk. In other words, will the project be on time and under budget, or will there be a schedule crash and budget overrun, and, if so, how bad can they be? To illustrate how quantitative risk management can be applied to



project management, we use ROV PEAT (available from Real Options Valuation, Inc. at www.realoptionsvaluation.com) to model these two sources of risks.

To follow along, start the PEAT software, select the Project Management–Dynamic Cost and Schedule Risk module, and Load Example. We begin by illustrating a simple linear path project in the Simple Linear Path 1 tab (Figure 3). Note that users can click on the Projects menu to add additional projects, or delete and rename existing projects. The example loaded has 5 sample predefined projects. In this simple linear path project, there are 11 sample tasks and each task is linked to its subsequent tasks linearly (i.e., Task 2 can only start after Task 1 is done, and so forth). For each project, a user has a set of controls and inputs:

- *Sequential Path versus Complex Network Path.* The first example illustrated uses the sequential path, which means there is a simple linear progression of tasks. In the next example, we will explore the complex network path where tasks can be executed linearly, simultaneously, and recombined at any point in time.
- *Fixed Costs.* The fixed costs and their ranges suitable for risk simulation (minimum, most likely, maximum) are required inputs. These fixed costs are costs that will be incurred regardless of there being an overrun in schedule (the project can be completed early or late but the fixed costs will be the same regardless).
- *Time Schedule.* Period-specific time schedule (minimum, most likely, maximum) in days, weeks, or months. Users will first select the periodicity (e.g., days, weeks, months, or unitless) from the droplist and enter the projected time schedule per task. This schedule will be used in conjunction with the variable cost elements (see next bullet item), and will only be available if *Include Schedule-Based Cost Analysis* is checked.
- *Variable Cost.* This is the variable cost that is incurred based on the time schedule for each task. This variable cost is per period and will be multiplied by the number of periods to obtain the total variable cost for



each task. The sum of all fixed costs and variable costs for all tasks will, of course, be the total cost for the project (denoted as *Project Total Cost*).

- *Overrun Assumption*. This is a percent budget buffer or cushion to include in each task. This column is only available and used if the *Include Budget Overruns and Buffers* checkbox is selected.
- *Probability of Success*. This allows users to enter the probability of each task being successful. If a task fails, then all subsequent tasks will be canceled and the costs will not be incurred, as the project stops and is abandoned. This column is available and will be used in the risk simulation only if the *Include Probabilities of Success* checkbox is selected.
- *Run*. The Run button will perform the relevant computations based on the settings and inputs, and also run risk simulations if the *Perform Risk Simulation* checkbox is selected (and if the requisite simulation settings such as distribution type, number of trials, and seed value settings are entered appropriately). This will run the current project's model. If multiple projects need to be run, you can first proceed to the *Portfolio Analysis* tab and click on the *Run All Projects* button instead.

To see which of these input assumptions drive total cost and schedule the most, a tornado analysis can be executed (Figure 4). The model can then be risk simulated and the results will show probability distributions of cost and schedule (Figure 5). For instance, the sample results show that for Project 1, there is a 95% probability that the project can be completed at a cost of \$398,594. The expected median or most likely value was originally \$377,408 (Figure 3). With simulation, it shows that to be 95% sure that there are sufficient funds to complete the project, an additional buffer of \$21,186 is warranted.



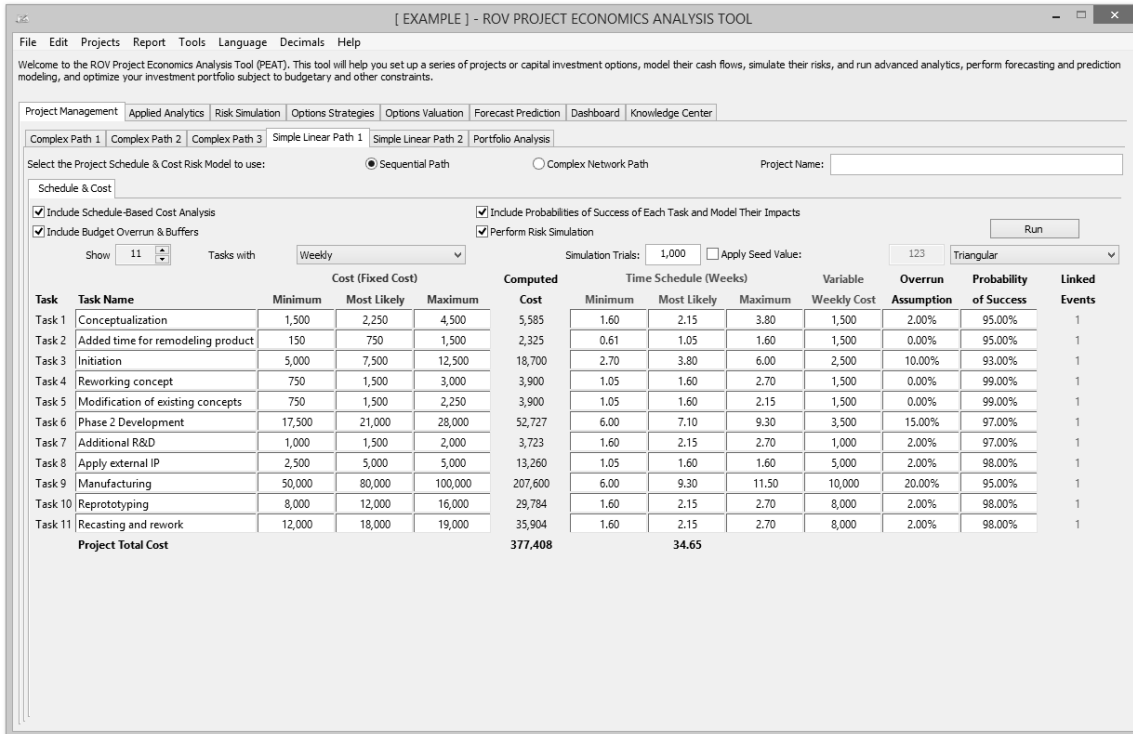


Figure 3: Simple Linear Path Project Management with Cost and Schedule Risk

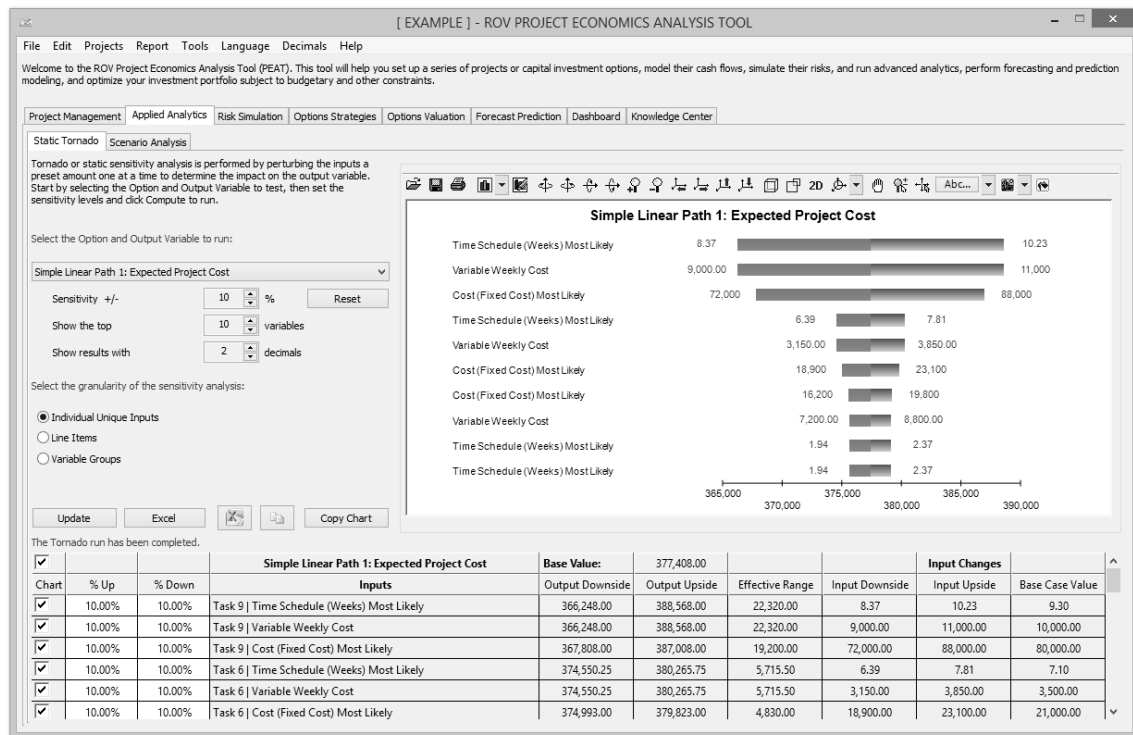


Figure 4: Simple Linear Path Tornado Analysis



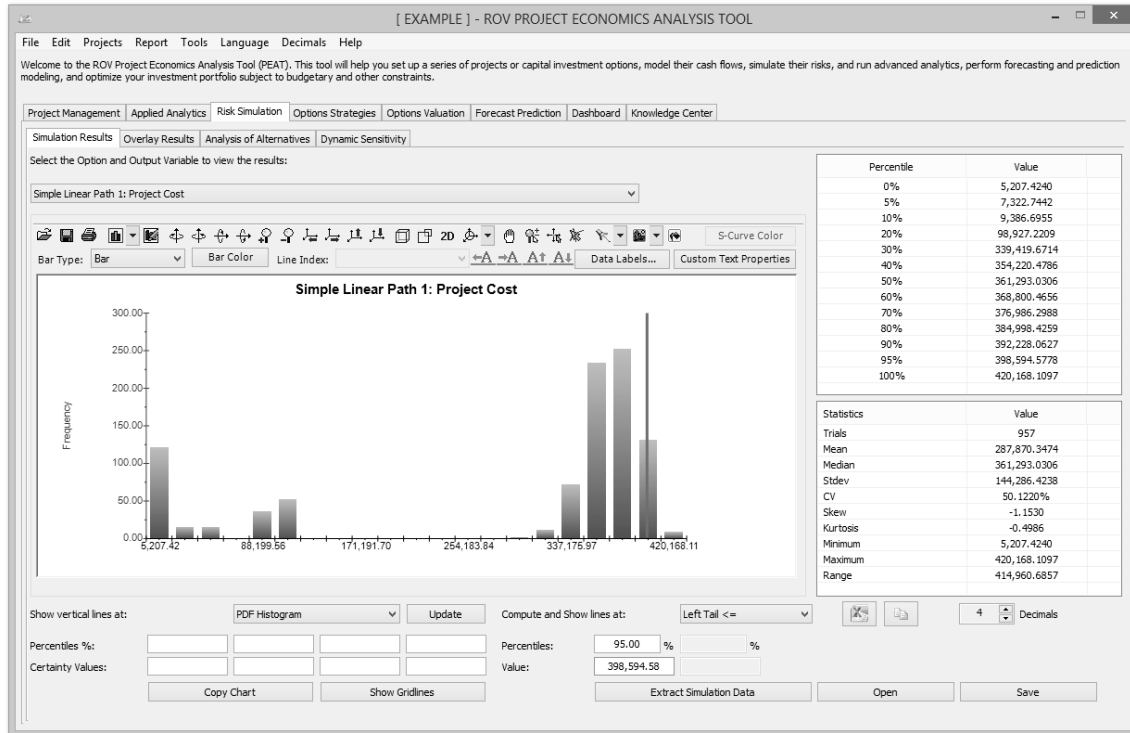


Figure 5: Monte Carlo Risk Simulated Results for Risky Cost and Schedule Values

In complex projects where there are nonlinear bifurcating and recombining paths (Figure 6), the cost and schedule risk is more difficult to model and compute. For instance, in the *Complex Path 1* tab, we can see that after Task 1, future tasks can be run in parallel (Tasks 2, 3, and 4). Then, Tasks 3 and 4 recombine into Task 8. Such complex path models can be created by the user simply by adding tasks and combining them in the visual map as shown. The software will automatically create the analytical financial model when *Create Model* is clicked. That is, you will be taken to the *Schedule & Cost* tab and the same setup as shown previously is now available for data entry for this complex model (Figure 7). The complex mathematical connections will automatically be created behind the scenes to run the calculations so that the user will only need to perform very simple tasks of drawing the complex network path connections. Below are some tips on getting started:

- Start by adding a new project from the *Projects* menu. Then, click on the *Complex Network Path* radio selection to access the *Network Diagram* tab.

- Use the icons to assist in drawing your network path. Hover your mouse over the icons to see their descriptions. You can start by clicking on the third icon to *Create a Task*, and then click anywhere in the drawing canvas to insert said task.
- With an existing task clicked on and selected, click on the fourth icon to *Add a Subtask*. This will automatically create the adjoining next task and next task number. You then need to move this newly inserted task to its new position. Continue with this process as required to create your network diagram. You can create multiple subtasks off a single existing task if simultaneous implementations occur.
- You can also recombine different tasks by clicking on one task, then holding down the Ctrl key and clicking on the second task you wish to join. Then click on the fifth icon to *Link Tasks* to join them. Similarly, you can click on the sixth icon to *Delete Link* between any two tasks.
- When the network diagram is complete, click on *Create Model* to generate the computational algorithms where you can then enter the requisite data in the *Schedule & Cost* tab as described previously.



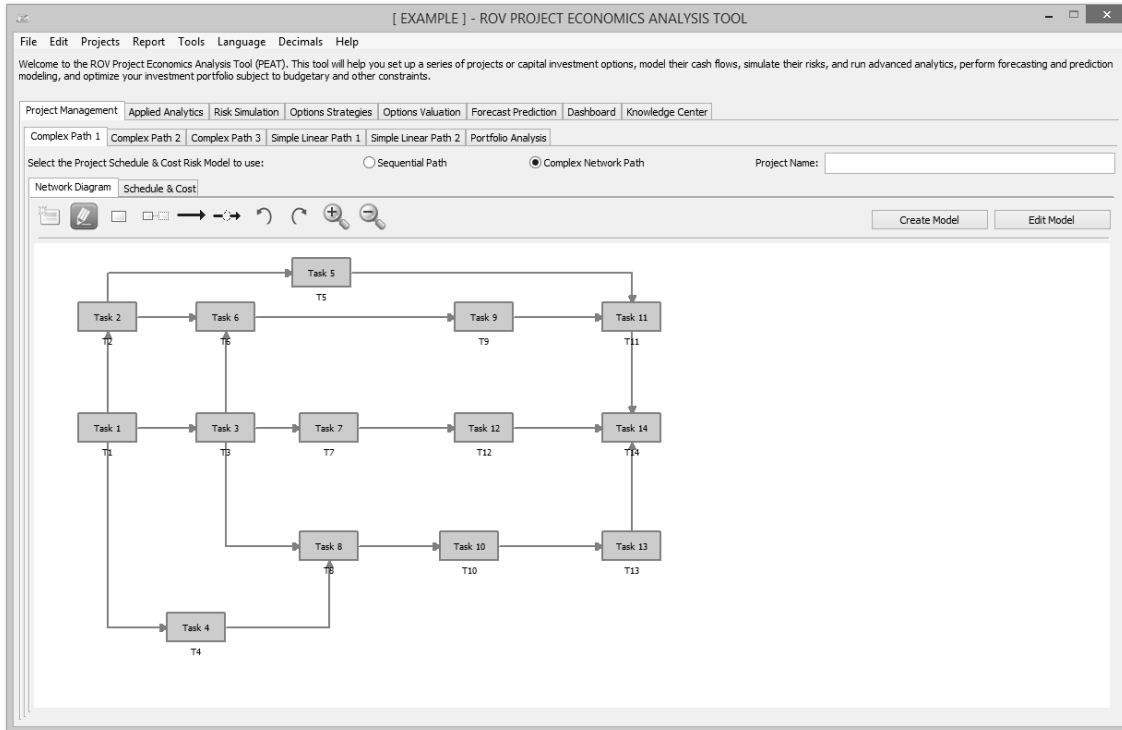


Figure 6: Complex Path Project Management

Task	Task Name	Cost (Fixed Cost)			Computed Cost	Time Schedule (Weeks)			Variable Weekly Cost
		Minimum	Most Likely	Maximum		Minimum	Most Likely	Maximum	
Task 1	T1	34	39	47	800	34	39	47	19.5
Task 2	T2	17	32	37	544	17	32	37	16
Task 3	T3	21	41	48	882	21	41	48	20.5
Task 4	T4	24	27	36	392	24	27	36	13.5
Task 5	T5	25	32	34	544	25	32	34	16
Task 6	T6	29	35	46	648	29	35	46	17.5
Task 7	T7	31	37	37	722	31	37	37	18.5
Task 8	T8	14	20	24	220	14	20	24	10
Task 9	T9	24	38	39	950	30	48	55	19
Task 10	T10	24	38	40	760	24	38	40	19
Task 11	T11	9	12	16	84	9	12	16	6
Task 12	T12	30	31	45	512	30	31	45	15.5
Task 13	T13	40	42	61	924	40	42	48	21
Task 14	T14	16	17	22	162	16	17	22	8.5
Project Total Cost					8,141				
Expected Total Duration						197.00			
Critical Path 1, 3, 8, 10, 13-14					56.30%				
Critical Path 1, 3, 6, 9, 11, 14					29.70%				
Critical Path 1, 4, 8, 10, 13-14					9.30%				

Figure 7: Complex Project Cost Simulated Cost and Duration Model with Critical Path



After running the model, the complex path map shows the highlighted critical path (Figure 8) of the project, that is, the path that has the highest potential for bottlenecks and delays in completing the project on time. The exact path specifications and probabilities of being on the critical path are seen in Figure 7 (e.g., there is a 56.30% probability that the critical path will be along Tasks 1, 3, 8, 10, 13, or 14).

If there are multiple projects or potential project path implementations, the portfolio view (Figure 9) compares all projects and implementation paths for the user to make a better and more informed risk-based decision. The simulated distributions can also be overlaid (Figure 10) for comparison.

Figure 9 allows users to see all projects that were modeled at a glance. Each project modeled can be either different projects or the same project modeled under different assumptions and implementation options (i.e., different ways of executing the project), to see which project or implementation option path makes more sense in terms of cost and schedule risks. The *Analysis of Alternatives* radio selected allows users to see each project as stand-alone (as compared to *Incremental Analysis* where one of the projects is selected as the base case and all other projects' results show their differences from the base case), in terms of cost and schedule: single-point estimate values, simulated averages, the probabilities each of the projects will have a cost or schedule overrun, and the 90th percentile value of cost and schedule. Of course, more detailed analysis can be obtained from the *Risk Simulation | Simulation Results* tab, where users can view all the simulation statistics and select any confidence and percentile values to show. The *Portfolio Analysis* tab also charts the simulated cost and schedule values using bubble and bar charts for a visual representation of the key results.



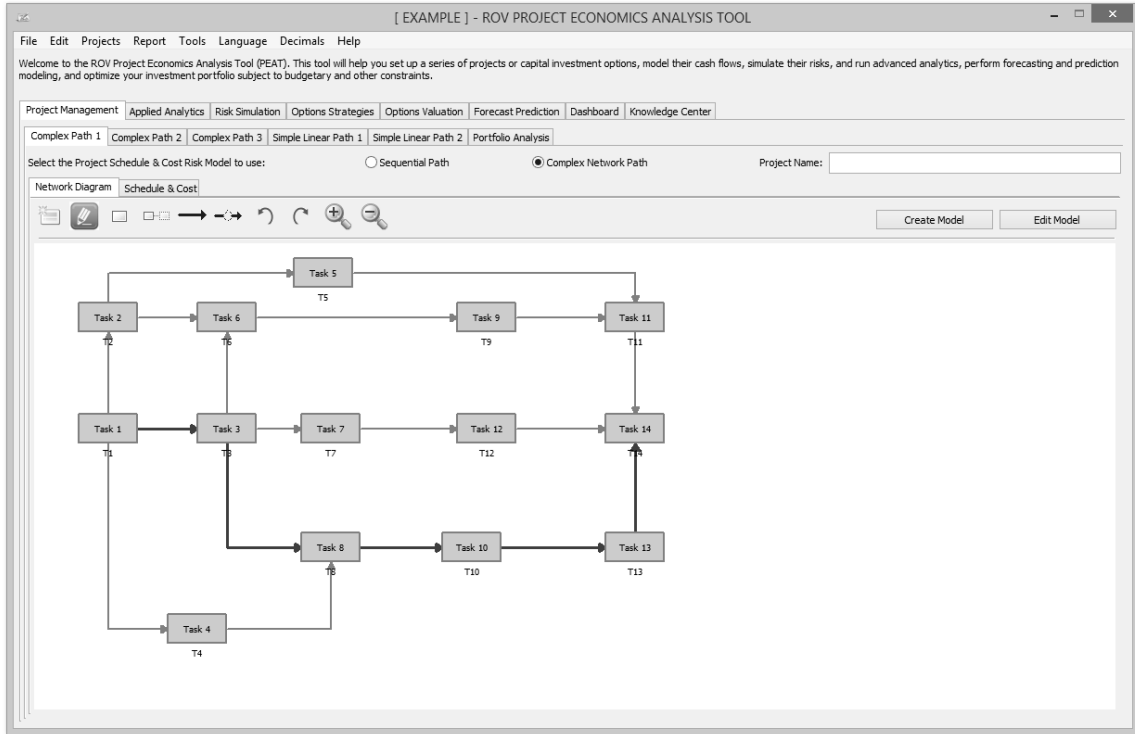


Figure 8: Complex Project Critical Path

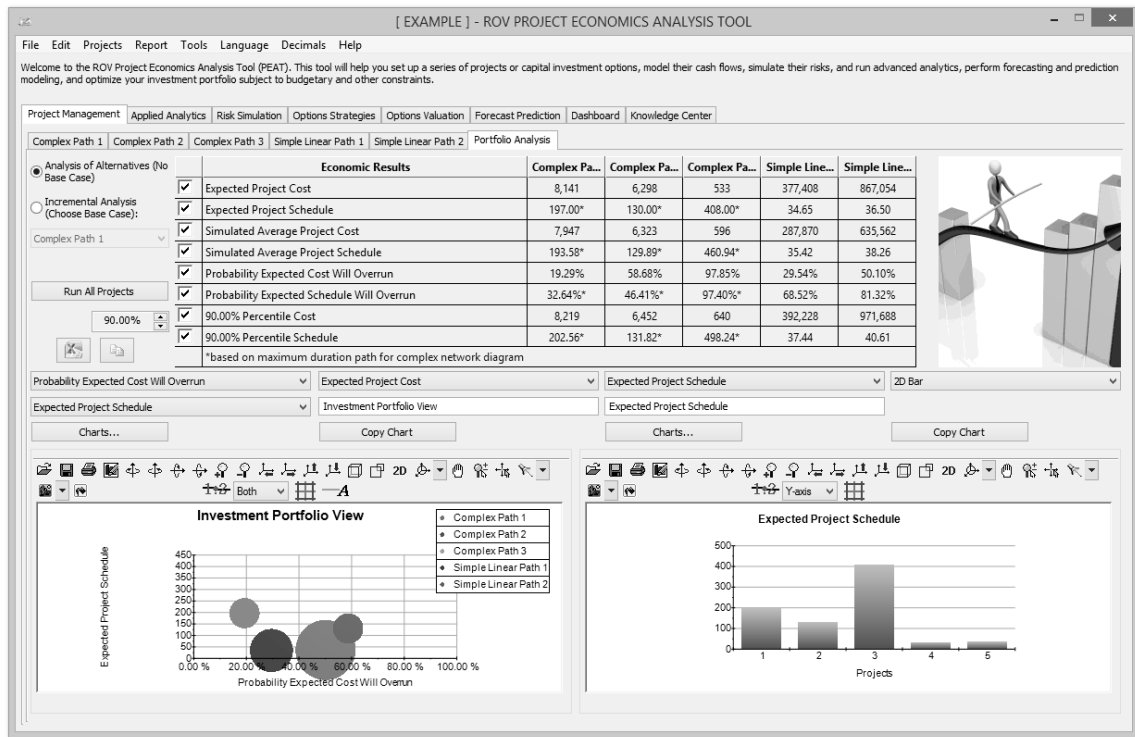


Figure 9: Portfolio View of Multiple Projects at Once

The *Overlay* chart in Figure 10 shows multiple projects' simulated costs or schedules overlaid on one another to see their relative spreads, location, and skew of the results. We clearly see that the project whose distribution lies to the right has a much higher cost to complete than the left, with the project on the right also having a slightly higher level of uncertainty in terms of cost spreads.

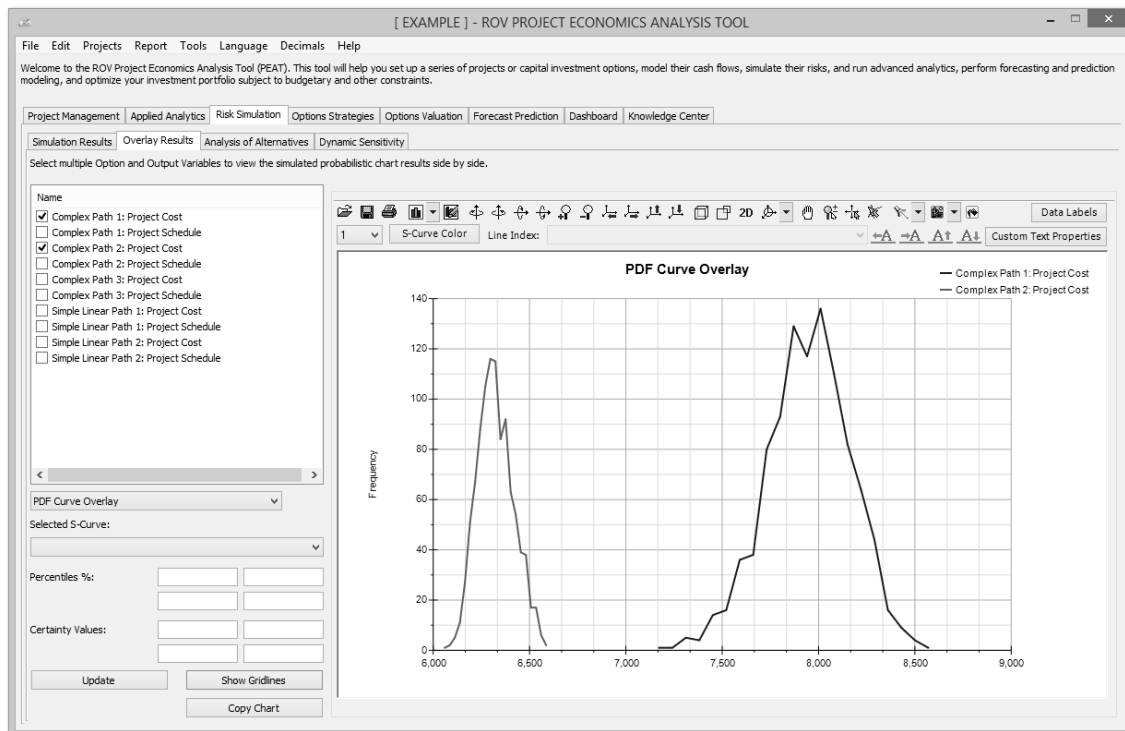


Figure 10: Overlay Charts of Multiple Projects' Cost or Schedule

Case Application: DDG 51 FLT III

This section details an illustration of the proposed integrated cost estimation modeling approach. As this is only an illustration, and due to a lack of proprietary data for this first phase of the analysis, the input assumptions are only high-level approximations based on publicly available information and subject matter expert estimates. Therefore, the results generated are not to be used in any specific decision making. Nonetheless, the approach presented is robust and valid, and with the correct input assumptions, can be rerun to generate accurate and reliable estimates. Information and data were obtained via publicly available sources, and were collected, collated, and used in an integrated risk-based cost and schedule modeling methodology. The objective of this study is to develop a comprehensive cost modeling strategy and approach, and as such, notional data were used. Specifically, we used the Arleigh Burke Class Guided Missile Destroyer DDG 51 Flight I, Flight II, Flight IIA, and Flight III (Figure 11) as a basis for the cost and schedule assumptions, but the modeling approach is extensible to any and all other ships within the U.S. Navy.

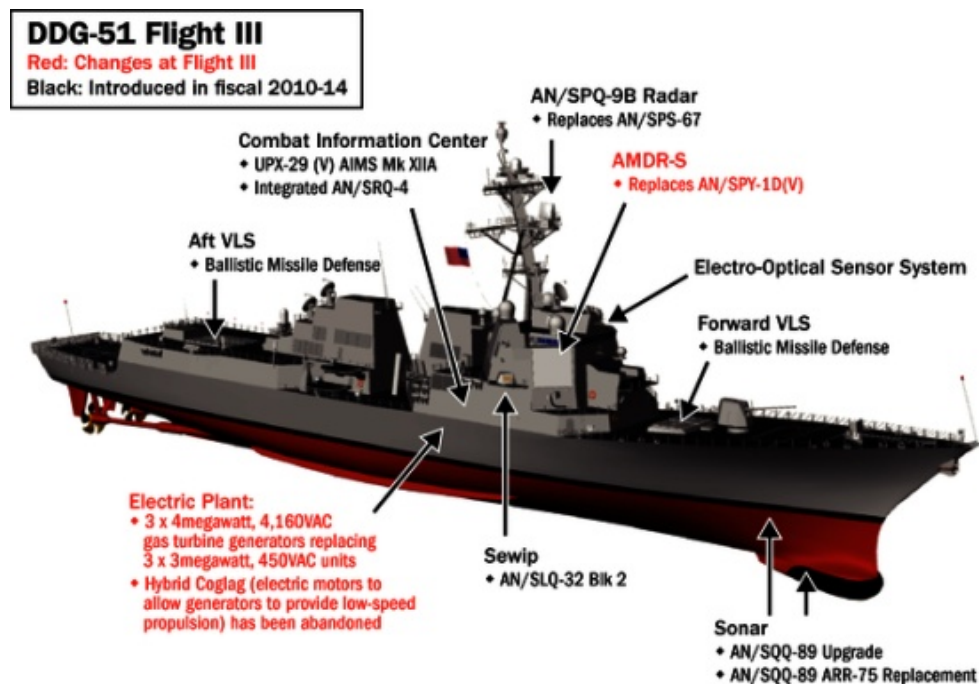


Figure 11: Overview of DDG 51 Flight III

Overview of the DDG 51 Arleigh Burke Destroyer

In the cost analysis models, we will consider the full build of the ship, with its accoutrements such as weapons systems, electrical systems, radar and electronic warfare systems, communication and navigation systems, aircraft, and other extra add-ons.

Figure 12 is a descriptive summary of the DDG 51 Arleigh Burke Destroyer. The DDG 51 is a guided missile destroyer in the U.S. Navy, with a complement of up to 96 missiles and a 5-inch gun for Naval Surface Warfare. The DDG 51 has multiple variants; in the current analysis we will consider the FLT III variant. One of the reasons the DDG 51 was selected for this analysis is because sufficient information on its acquisitions process is available since two DDG 51 AEGIS destroyers have been funded in FY 2016. These two ships are part of a 10-ship procurement between FY 2013 and FY 2017.

FY 2016 Program Acquisition Costs by Weapon System

DDG 51 ARLEIGH BURKE Class Destroyer

USN

The DDG 51 class guided missile destroyers provide a wide range of warfighting capabilities in multi-threat air, surface, and subsurface environments.

The DDG 51 class ship is armed with a vertical launching system, which accommodates 96 missiles, and a 5-inch gun that provides Naval Surface Fire Support to forces ashore and anti-ship gunnery capability against other ships. The DDG 51 class is the first class of destroyers with a ballistic missile defense capability.

The Arleigh Burke class is comprised of four separate variants; DDG 51-71 represent the original design, designated Flight I ships, and are being modernized to current capability standards; DDG 72-78 are Flight II ships; DDG 79-123 ships are Flight IIA ships; and, in FY 2016, DDG-124 will become the first Flight III ship. Flight III ships will feature the Air and Missile Defense Radar (AMDR) capability.

Mission: Provides air and maritime dominance and land attack capability with its AEGIS Weapon System, AN/SQQ-89 Anti-Submarine Warfare System, and Tomahawk Weapon Systems.

FY 2016 Program: Funds two DDG 51 AEGIS class destroyers as part of a multiyear procurement for ten ships from FY 2013 - FY 2017.



Figure 12: DDG 51 Specifications

DoD Spending on the Aegis Destroyer in FY 2012–2014

Figure 13 shows some sample acquisition budgets for DDG 51 Aegis destroyers from FY 2012 through FY 2016. The comprehensive DoD budget was downloaded and analyzed in the current research.

DoD Spending, Procurement and RDT&E: FY 2012/13/14 + Budget for FYs 2015 + 2016 [Go to Top](#)

DDG 51 AEGIS Destroyer	ACTUAL		ACTUAL		ACTUAL		PRELIMINARY		REQUESTED		FY2017-FY2020 Budget Data In PDF files below table → → →
	FY2012 Total	FY2013 Total	FY2014 Total	FY2015 Total	FY2016 Total	QTY	Million \$	QTY	Million \$		
<i>Procurement</i>											
Shipbuilding & Conversion	NAVY	1 2,081.43	3 4,497.01	1 1,985.12	2 2,795.95	2	2,795.95	2	3,149.70		
Ship Modifications	NAVY	126.37	407.71	285.99	324.22		324.22		364.16		
Completion Costs	NAVY	-	-	100.00	129.14		129.14		-		
Outfitting & Post Delivery	NAVY	49.10	7.30	1.30	6.50		6.50		62.10		
Total Procurement		1 2,256.91	3 4,912.02	1 2,372.41	2 3,255.81	2	3,255.81	2	3,575.96		
RDT&E (Hybrid Electric Drive)	NAVY	-	-	-	7.95		7.95		4.22		
Total RDT&E		-	-	-	7.95		7.95		4.22		
Total Program Spending		1 2,256.91	3 4,912.02	1 2,372.41	2 3,263.76	2	3,263.76	2	3,580.18		

Download Official U.S. Department of Defense (DoD) Budget Data:

[Shipbuilding & Conversion | DDG-51 AEGIS Destroyer](#)

Figure 13: DoD Spending and Procurement for FY 2012–2014

High-Level Shipbuilding Process

Figure 14 shows the high-level process flow of building ship hulls and sections.



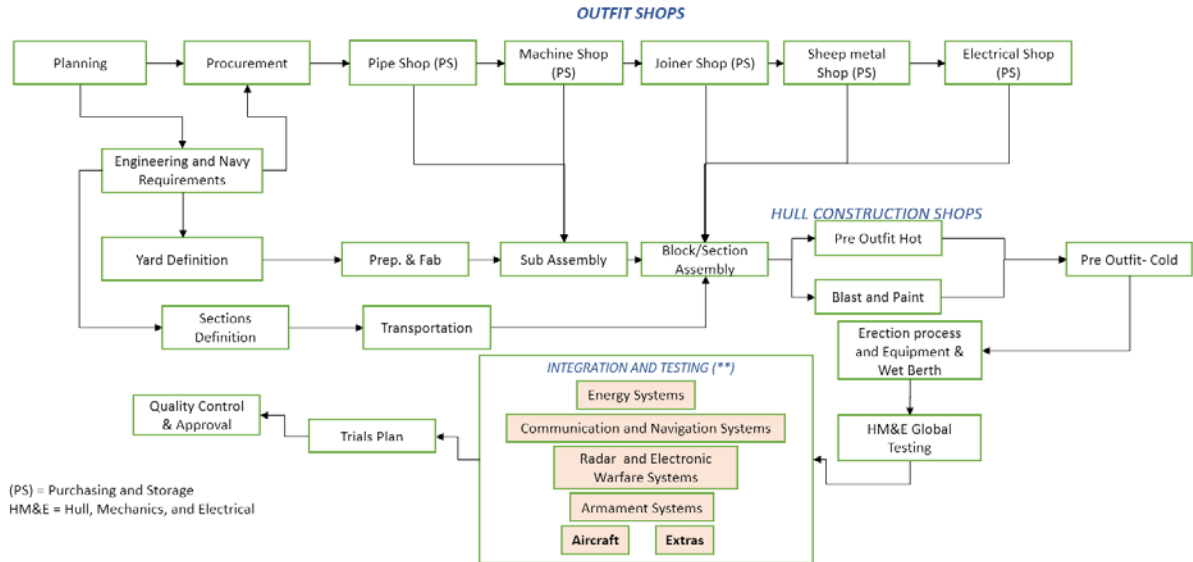


Figure 14: High-Level Process Flow (Hull and Sections)

Information, Communication, and Technology Subprocess

Figure 15 shows the ship's subprocess for information, communication, and technology (ICT).

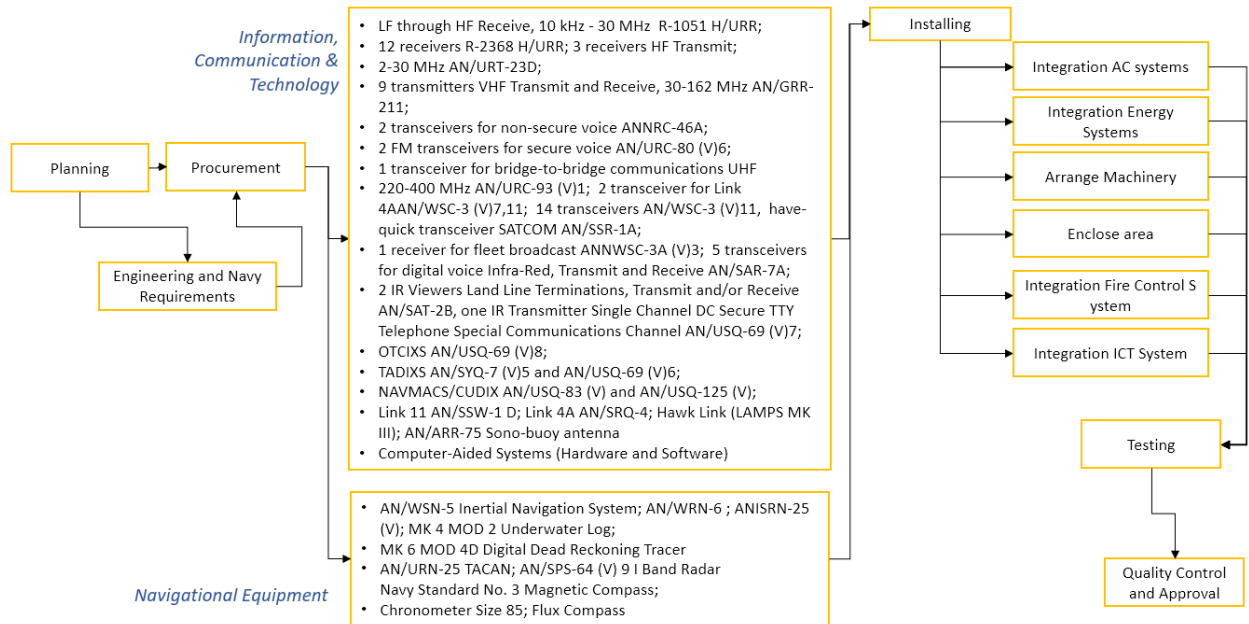


Figure 15: Subprocess for Information, Communication, and Technology (ICT)



Weapons System Subprocess

Figure 16 shows the ship's subprocess for weapons systems.

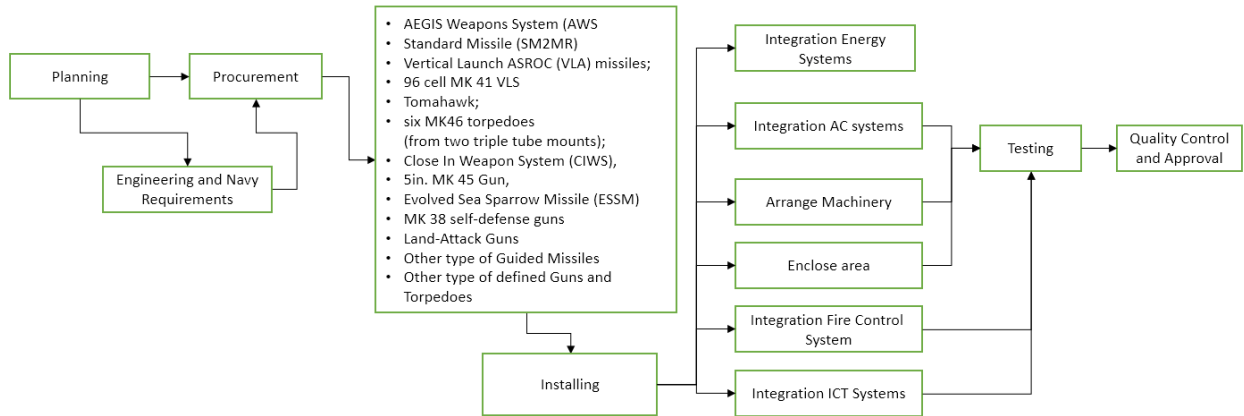
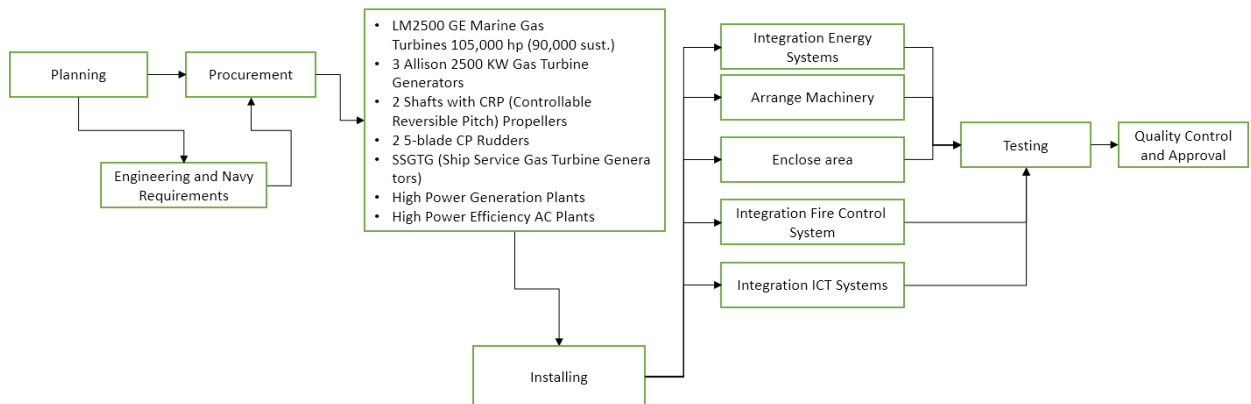


Figure 16: Subprocess for Weapons Systems

Electrical Systems Subprocess

Figure 17 shows the ship's electrical systems subprocess.

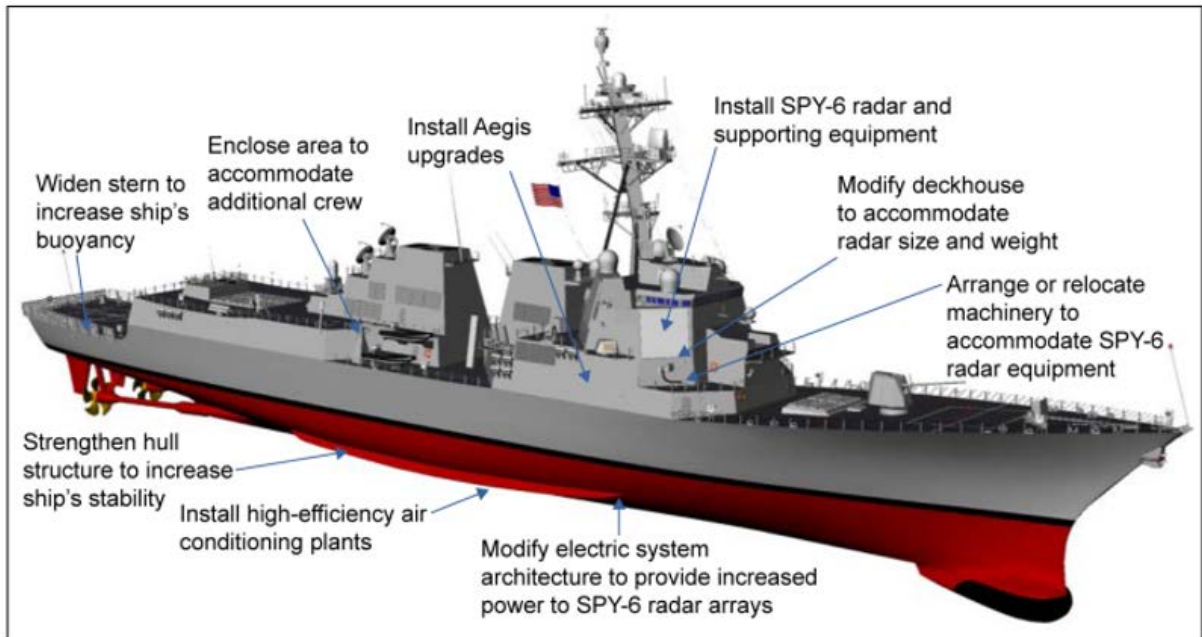


Propulsion: General Electric LM2500 gas turbines each generating 26,500 HP (19,800 kW); coupled to two shafts, each driving a five-bladed reversible controllable-pitch propeller

Figure 17: Subprocess for Electrical Systems

SPY-6 Radar System

Figure 18 shows the ship's radar subsystem's process.



Source: GAO (analysis); Navy (image and data). | GAO-16-613

ARLEIGH BURKE DESTROYERS:

Delaying Procurement of DDG 51 Flight III Ships Would Allow Time to Increase Design Knowledge

GAO-16-613: Published: Aug 4, 2016. Publicly Released: Aug 4, 2016.

What GAO Found

The Air and Missile Defense Radar (AMDR) program's SPY-6 radar is progressing largely as planned, but extensive development and testing remains. Testing of the integrated SPY-6 and full baseline Aegis combat system upgrade—beginning in late 2020—will be crucial for demonstrating readiness to deliver improved air and missile defense capabilities to the first DDG 51 Flight III ship in 2023. After a lengthy debate between the Navy and the Department of Defense's (DOD) Director of Operational Test and Evaluation, the Secretary of Defense directed the Navy to fund unmanned self-defense test ship upgrades for Flight III operational testing, but work remains to finalize a test strategy.

Figure 18: SPY-6 Radar System and Rework

DoD Extras: Electronic Warfare, Decoys, Extra Capabilities

Figure 19 shows the ship's Electronic Warfare, Decoys, and Extra Capabilities subprocesses.

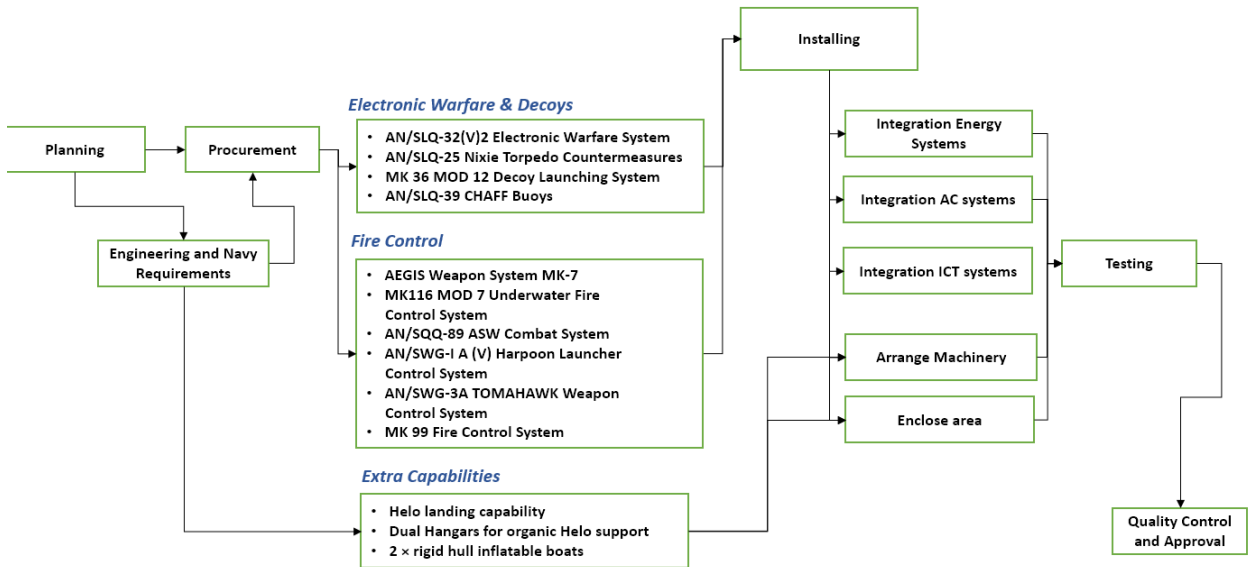


Figure 19: Subprocesses and Examples of DoD Extras

Risk-Based Schedule and Cost Process Modeling

Figure 20 illustrates how the project management tasks are incorporated into the PEAT software application. The parallel development of tasks 20–25 is where the ship's various subsystems are incorporated into the cost and schedule analysis.

Further, Figures 21, 22, and 23 show how some of the publicly available data are collated and incorporated as assumptions into the PEAT software (Figure 24).

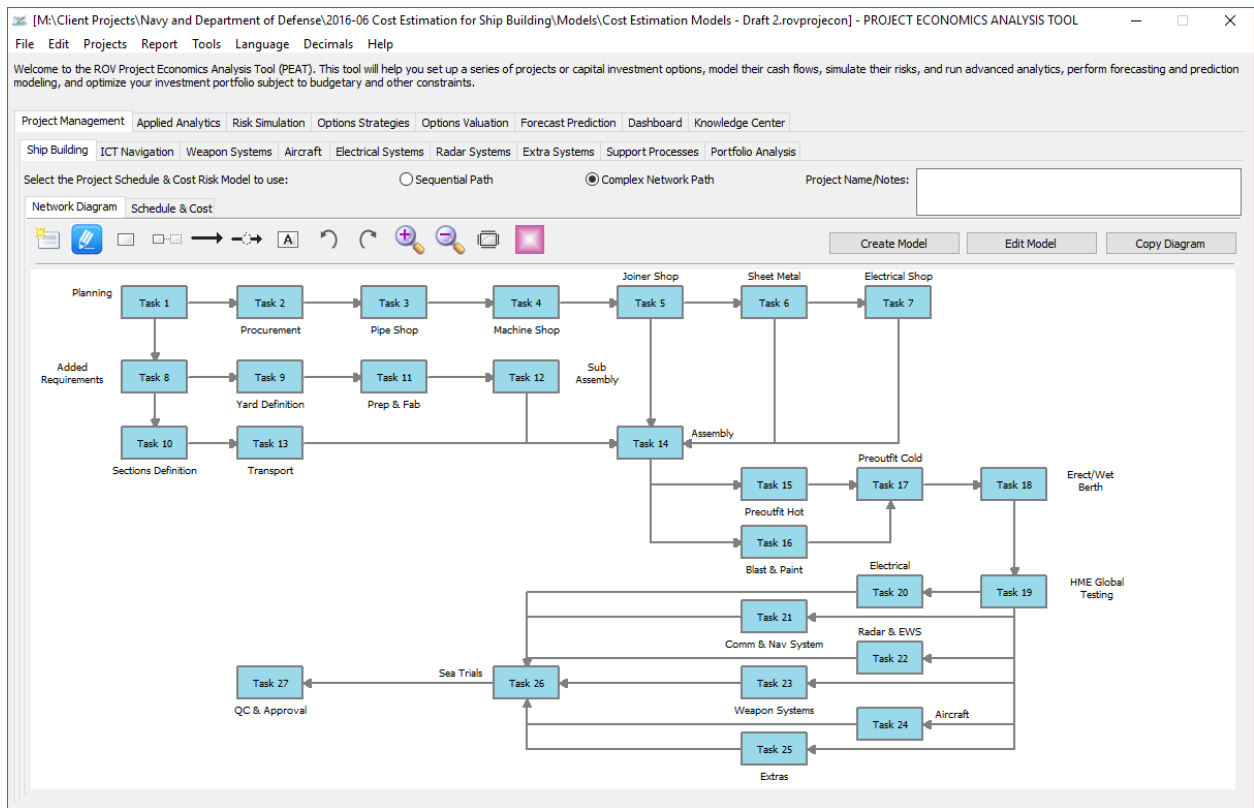


Figure 20: Modeling Overall Process

Items	Quantity	Min Unit Cost	Aveg Unit Cost	Max Unit Cost	Total Cost (\$M)
Interior Communications	5.00	967.39	1,248.62	1,897.64	1,248.62
AN/STC-2(V) Integrated Voice Communications System (IVCS), IC	1	0.0064	0.008	0.0096	0.01
AN/USQ-82(V) Fiber Optic Data Multiplex System (FODMS)	1	0.784	0.98	1.176	0.98
Exterior Communications:					
High Frequency (HF) radio group AN/URC-131A(V)	2	0.012	0.015	0.018	0.03
Very High Frequency (VHF) transmit and receive, 30-162 MHz:					
- AN/GRC-211; two transceivers for non-secure voice.					
- AN/VRC-46A; two FM transceivers for secure voice.					
- AN/URC-139; one transceiver for bridge-to-bridge communications.	3	0.008	0.01	0.012	0.03
Ultra High Frequency (UHF) transmit and receive, 220-400 MHz:					
- AN/GRC-171B(V)4; two transceivers for Link 4A.					
- AN/WSC-3(V)7, 11; sixteen transceivers.	2	0.044	0.062	0.08	0.12
Satellite Communications (SATCOM) transmit and/or receive:					
- AN/USQ-122A(V); one receiver for fleet broadcast.					
- AN/WSC-3(V)15; two transceivers for digital exchange system.	2	0.04	0.057	0.074	0.11
38(V)2; one transceiver.	1	1.009	1.262	1.514	1.26
Infrared transmit and receive:					
- AN/SAT-2A; one IR transmitter.					
Landline terminations, transmit and/or receive:					
- Single channel Disable Communications (DC) secure Teletypewriter (TTY).					
- Telephone.					
Special communications channel:					
- ON-143(V)6/USQ: Officer in Tactical Command Information Exchange Subsystem(OTCIXS).					
- ON-143(V)6/USQ: Tactical Data Information Exchange System (TADIXS).					
- TADIXS-B/CTT-H3.					
- AN/SYQ-7A(V): Naval Modular Automated Communication System/Common User Digital Exchange System (NAVMACS/CUDIXS).					
- AN/UYQ-62(V)2, Command and Control Processor (C2P).	7	0.04	0.056	0.072	0.39
Underwater Communications:					
- AN/WQC-2A sonar communications set.					
- AN/WQC-6 sonar communications set.	2	4.133	5.166	6.199	10.33

Figure 21: Cost information on Communications and Radar Systems

Category	Items	Quantity	Min Unit Cost	Aveg Unit Cost	Max Unit Cost	Total Cost (\$M)
Navigational Equipment	AN/WNS-5 Inertial Navigation System; AN/WRN-6 ; ANISRN-25 (V); MK 4 MK 6 MOD 4D Digital Dead Reckoning Tracer	1	8	14	20	14.00
	AN/URN-25 TACAN; AN/SPS-64 (V) 9 I Band Radar Navy Standard No. 3 Magnetic Compass; Total Navigation system Chronometer Size 85; Flux Compass	1	15.84	19.8	23.76	19.80
	Total	2	23.84	33.80	43.76	33.80
Weapons	RIM-66 Standard Missile SM-2MR; RIM-67/RIM-156 Standard Missile SM-2ER					
	RIM-161 Standard Missile SM-3	74	3	3.24	10.07	239.76
	Vertical Launch ASROC (VLA) missiles;					
	MK 41 Vertical Missile Launch Systems (VLS)	2	38.2	110.1	182	220.20
	BGM-109 Tomahawk	1	0.4552	0.569	0.6828	0.57
	MK-46 torpedoes (from two triple tube mounts);	6				
	Close In Weapon System (CIWS), MK-45 (Mod.1/2) 5"/54	1	3.04	3.8	4.56	3.80
	RIM Evolved Sea Sparrow Missile (ESSM)	1	0.84	0.905	0.97	0.91
	MK 38 self--defense guns Land-Attack Guns					
	Other type of Guided Missiles (Guided shell)	10	0.025	0.0375	0.05	0.38
Other type of defined Guns and Torpedoes, missiles, being part of the ship's	1	641.40344	796.77	1296.242	796.77	
	Total	96	686.96	915.42	1494.57	1262.38
Aircraft	MH-60 B/R Seahawk LAMPS III helicopters with Penguin/ Hellfire missiles	2	27.693	30.77	60	61.54
	MK 46/MK 50 torpedoes					

Figure 22: Cost Information on Navigation, Weapons, and Aircraft Systems



Category	Items	Quantity	Min Unit Cost	Aveg Unit Cost	Max Unit Cost	Total Cost (\$M)
Energy Systems	LM2500 GE Marine Gas Turbines 105,000 shp (90,000 sust.)	4	2.000	2.5	3.000	10.00
	3 Allison 2500 KW Gas Turbine Generators	3	0.280	0.35	0.420	1.05
	2 Shafts with CRP (Controllable Reversible Pitch) Propellers	2				
	2 5-blade CP Rudders	2				
	SSGTG (Ship Service Gas Turbine Generators)	1				
	High Power Generation Plants	1				
	High Power Efficiency AC Plants	1				
Total		14.00	2.28	2.85	3.42	11.05
Radar Systems	AN/SPY-6(V) Air and Missile Defense Radar (AMDR)					
	Air & Missile Defense Radar (A&MD Radar) and Combat System Integrator	1	308.560	385.70	462.840	385.70
Total		1.00	308.56	385.70	462.84	385.70
Electronic warfare & decoys	AN/SLQ-32(V)2 Electronic Warfare System	1	2.000	2.5	3.000	2.50
	AN/SLQ-25 Nixie Torpedo Countermeasures					
	MK 36 MOD 12 Decoy Launching System					
	AN/SLQ-39 CHAFF Buoys					
Total		1.00	2.00	2.50	3.00	2.50
Fire control	AEGIS Weapon System MK-7	1	51.240	42.7	51.240	42.70
	MK116 MOD 7 Underwater Fire Control System					
	AN/SQQ-89 ASW Combat System					
	AN/SWG-I A (V) Harpoon Launcher Control System					
	AN/SWG-3A TOMAHAWK Weapon Control System					
	MK 99 Fire Control System					
Total		1.00	51.24	42.70	51.24	42.70
Extra capabilities	Helo landing capability					
	Dual Hangars for organic Helo support					
	Rigid hull inflatable boats (Defender)	2	0.01	0.0175	0.025	0.04
Total		2.00	0.01	0.02	0.03	0.04
Support	Support services and Yard Admin	1	7	10	20	10.00
	Total	1.00	7.00	10.00	20.00	10.00

Figure 23: Cost Information on Electrical, Electronic Warfare, Fire Control, and Additional Systems

Task	Task Name	19.70	24.62	29.55	27	1.27	2.00	2.73	0.04	10.00%
Task 6	Sheet Metal	19.70	24.62	29.55	27	1.27	2.00	2.73	0.04	10.00%
Task 7	Electrical Shop	19.70	24.62	29.55	29	3.17	5.00	6.84	0.40	10.00%
Task 8	Added Requirements	2.36	3.07	4.76	4	2.53	4.00	5.47	0.16	10.00%
Task 9	Yard Definition	2.63	3.41	5.29	4	2.53	4.00	5.47	0.16	10.00%
Task 10	Sections Definition	2.89	3.75	5.82	4	1.27	2.00	2.73	0.16	10.00%
Task 11	Prep & Fab	1.84	2.38	3.70	4	3.80	6.00	8.20	0.16	10.00%
Task 12	Sub Assembly	21.01	27.25	42.33	31	2.53	4.00	5.47	0.24	10.00%
Task 13	Transport	13.13	17.03	26.45	20	1.90	3.00	4.10	0.24	10.00%
Task 14	Assembly	31.51	40.88	63.49	47	3.17	5.00	6.84	0.40	10.00%
Task 15	Preoutfit Hot	13.13	17.03	26.45	20	1.90	3.00	4.10	0.24	10.00%
Task 16	Blast & Paint	3.15	4.09	6.35	5	1.90	3.00	4.10	0.24	10.00%
Task 17	Preoutfit Cold	2.63	3.41	5.29	4	1.27	2.00	2.73	0.16	10.00%
Task 18	Erect/Wet Berth	39.39	51.10	79.36	57	1.90	3.00	4.10	0.24	10.00%
Task 19	HME Global Testing	55.14	71.54	111.10	87	6.33	10.00	13.67	0.79	10.00%
Task 20	Electrical	4.40	11.05	17.70	20	17.07	44.00	70.93	0.16	10.00%
Task 21	Comm & Nav System	19.64	47.07	74.50	61	19.40	50.00	80.60	0.16	10.00%
Task 22	Radar & EWS	158.16	385.70	613.24	435	23.28	60.00	96.72	0.16	10.00%
Task 23	Weapon Systems	514.54	1,262.38	2,010.21	1,397	18.62	48.00	77.38	0.16	10.00%
Task 24	Aircraft	24.56	61.54	98.52	71	13.97	36.00	58.03	0.08	10.00%
Task 25	Extras	18.03	45.24	72.44	52	9.31	24.00	38.69	0.08	10.00%
Task 26	Sea Trials	42.01	54.50	84.65	74	5.06	8.00	10.94	1.59	10.00%
Task 27	QC & Approval	26.26	34.07	52.91	38	1.90	3.00	4.10	0.24	10.00%

Figure 24: Input Assumptions



Similarly, using the cost and schedule modeling approach, we can zoom into various tasks and model each task in more detail, and using the results, reinsert the values back into the more comprehensive model as required. For instance, Figure 25 shows the ship's weapons subsystem, with Figure 26 showing its cost and schedule assumptions. This model's result can be inserted back into Task 23 in the comprehensive model (Figure 20).

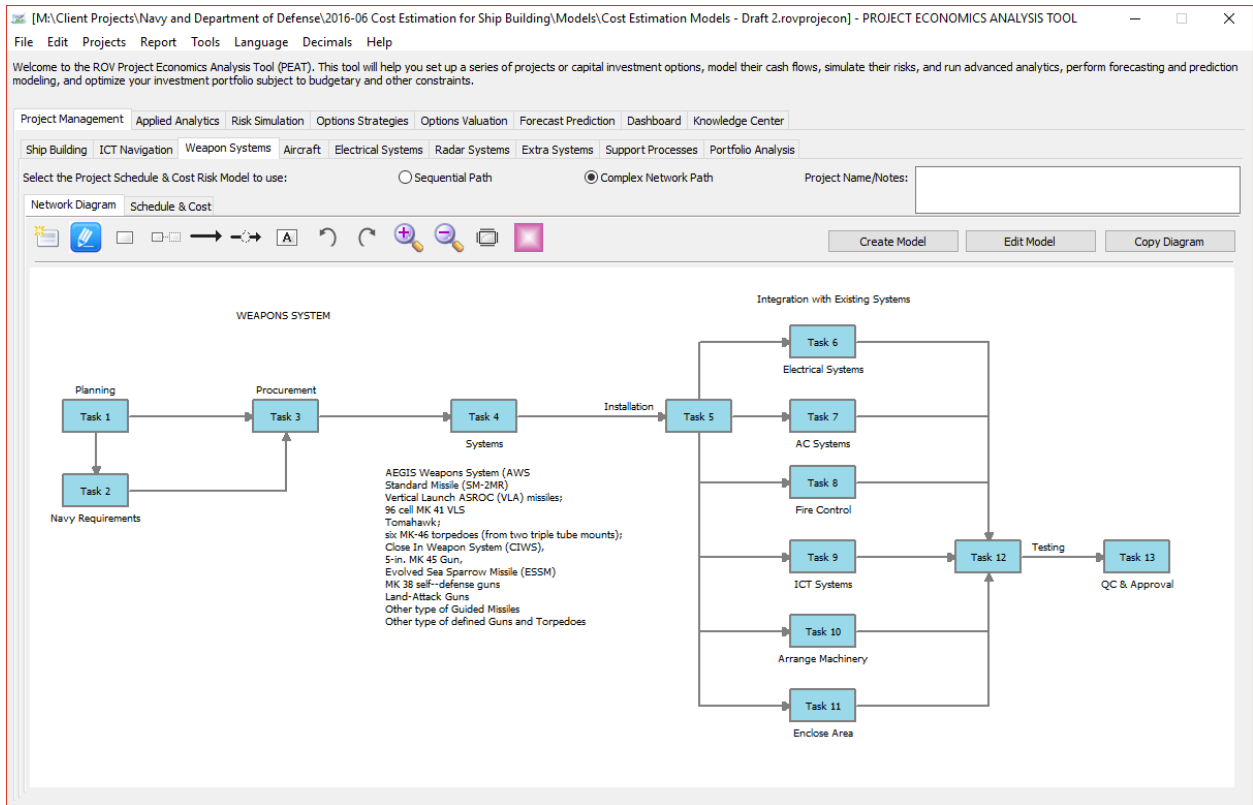


Figure 25: Weapons System Process Development



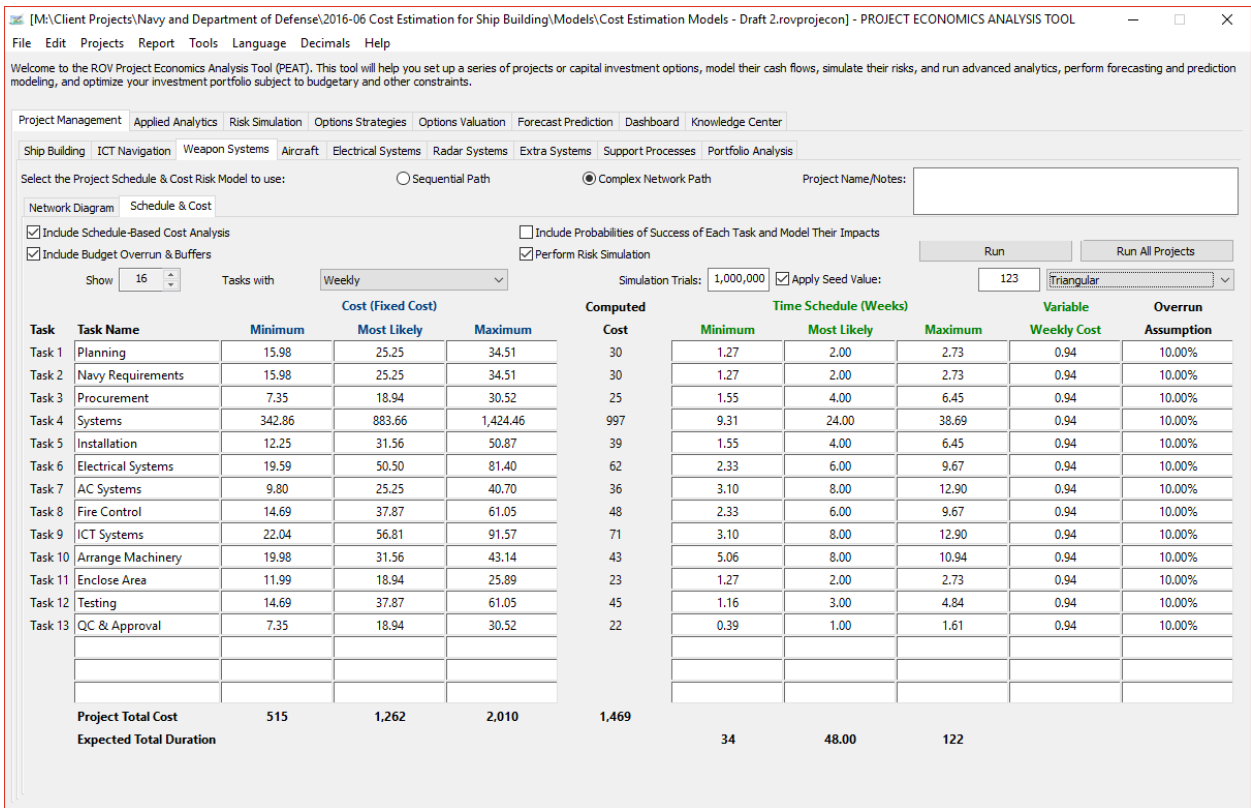


Figure 26: Weapons Subsystem Cost and Schedule Assumptions

Critical Success Factors in Cost and Schedule Estimates

Tornado analysis is a powerful analytical tool that captures the critical success factors via identifying the static impacts of each variable on the outcome of the model; that is, the tool automatically perturbs each variable in the model a preset amount, captures the fluctuation on the model's forecast or final result, and lists the resulting perturbations ranked from the most significant to the least. Figures 27 and 28 illustrate the application of a tornado analysis. Tornado analysis answers the question "What are the critical success drivers that affect the model's output the most?"



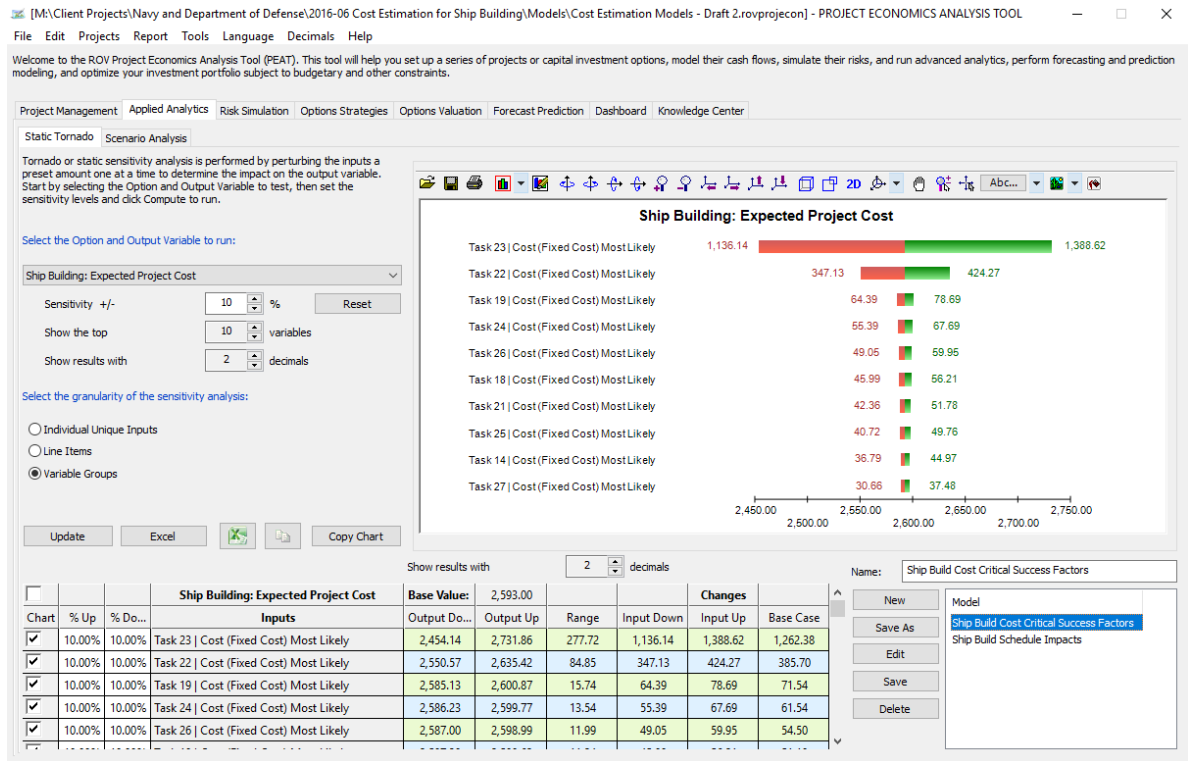


Figure 27: Tornado Analysis of Critical Success Factors (Cost Factors)

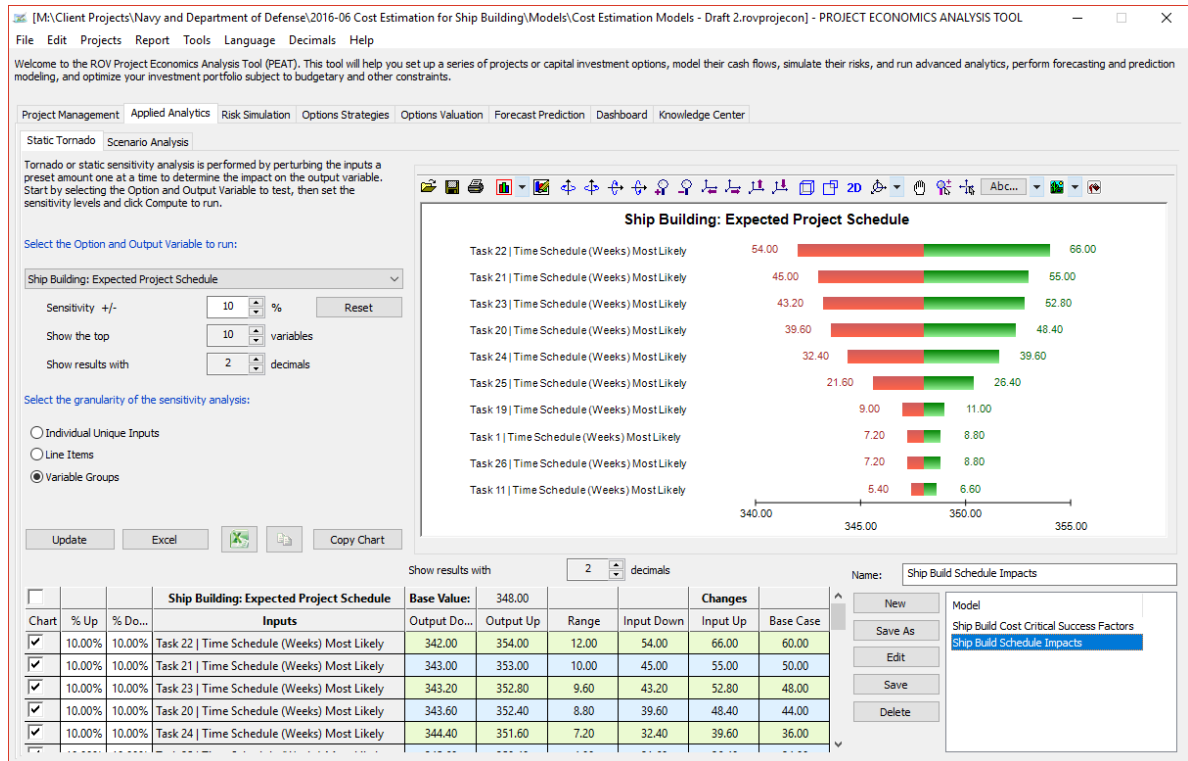


Figure 28: Tornado Analysis of Critical Success Factors (Schedule Factors)



Risk-Based Schedule and Cost Process Simulation

Next, Monte Carlo Risk Simulation was used to create artificial futures by generating hundreds of thousands of sample paths of outcomes and analyzing their prevalent characteristics. In the Monte Carlo simulation process, triangular distributions were used on the previously identified critical inputs. Figure 29 shows the values for a sample distributional spread used in Monte Carlo Risk Simulations per the *Air Force Cost Analysis Handbook (AFCAH)*. These probability spreads were applied to each of the task's cost and schedule inputs, and each of the tasks were simulated tens of thousands to hundreds of thousands of trials.

Figure 30 shows a sample representation of the results from the simulation process. For instance, the 90% confidence interval for the total acquisition cost of a full-complement ship (fully built ship delivered after tests and sea trials, complete with ICTS, weapons systems, electrical systems, SPY-6 radar, and other add-ons). The 90% confidence interval pegs the total acquisition costs to be between \$2.0 billion and \$3.2 billion for a single ship. Clearly, these results are only for illustration purposes and are not meant to be definitive. Figure 31 shows the probability that there will be a budget overrun. For instance, if the acquisition budget is \$2.2 billion, then we see that there is an approximately 12% probability of the cost coming in at or under budget, which means that there is an 88% probability of a budget overrun, with a mean or average actual acquisition cost of \$2.6 billion.

Similarly, Figure 32 shows the total schedule from the initial contracting phase to delivery of the ship, complete with all subsystems installed and tested. The 90% confidence interval pegs the total schedule at between 110 and 146 weeks, with an average of 127 weeks.



U.S. Air Force Cost Analysis Handbook (AFAH)

Distribution	PEI	Probability	Fitted Distributions					
			15%	Mode	85%	Min	Likely	Max
Triangular Low Left	Mode	1.0 (75%)	0.695	0.878	1.041	0.482	0.878	1.247
Triangular Low	Mode	1.0 (50%)	0.834	1	1.166	0.633	1.000	1.367
Triangular Low Right	Mode	1.0 (25%)	0.959	1.122	1.305	0.753	1.122	1.518
Triangular Medium Left	Mode	1.0 (75%)	0.492	0.796	1.069	0.137	0.796	1.412
Triangular Medium	Mode	1.0 (50%)	0.723	1	1.277	0.388	1.000	1.612
Triangular Medium Right	Mode	1.0 (25%)	0.931	1.204	1.508	0.588	1.204	1.863
Triangular High Left	Mode	1.0 (75%)	0.347	0.754	1.103	0.000	0.754	1.550
Triangular High	Mode	1.0 (50%)	0.612	1	1.388	0.142	1.000	1.858
Triangular High Right	Mode	1.0 (25%)	0.903	1.236	1.711	0.442	1.236	2.225
Triangular EHigh Left	Mode	1.0 (75%)	0.3	0.745	1.15	0.000	0.745	1.657
Triangular EHigh	Mode	1.0 (50%)	0.509	1.004	1.5	0.000	1.004	2.100
Triangular EHigh Right	Mode	1.0 (25%)	0.876	1.367	1.914	0.258	1.367	2.553

Figure 29: Sample Distributional Spread per the U.S. Air Force Cost Analysis Handbook

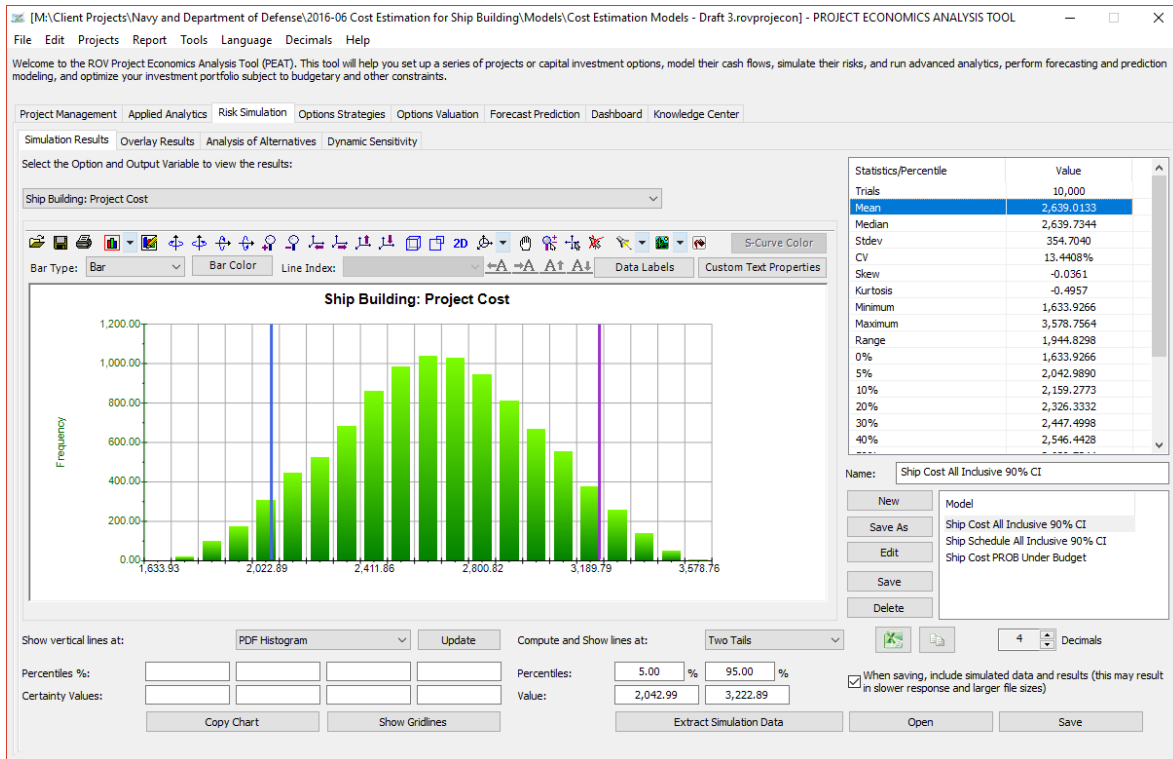


Figure 30: Simulation Results on Shipbuilding Cost (90% Confidence Interval)



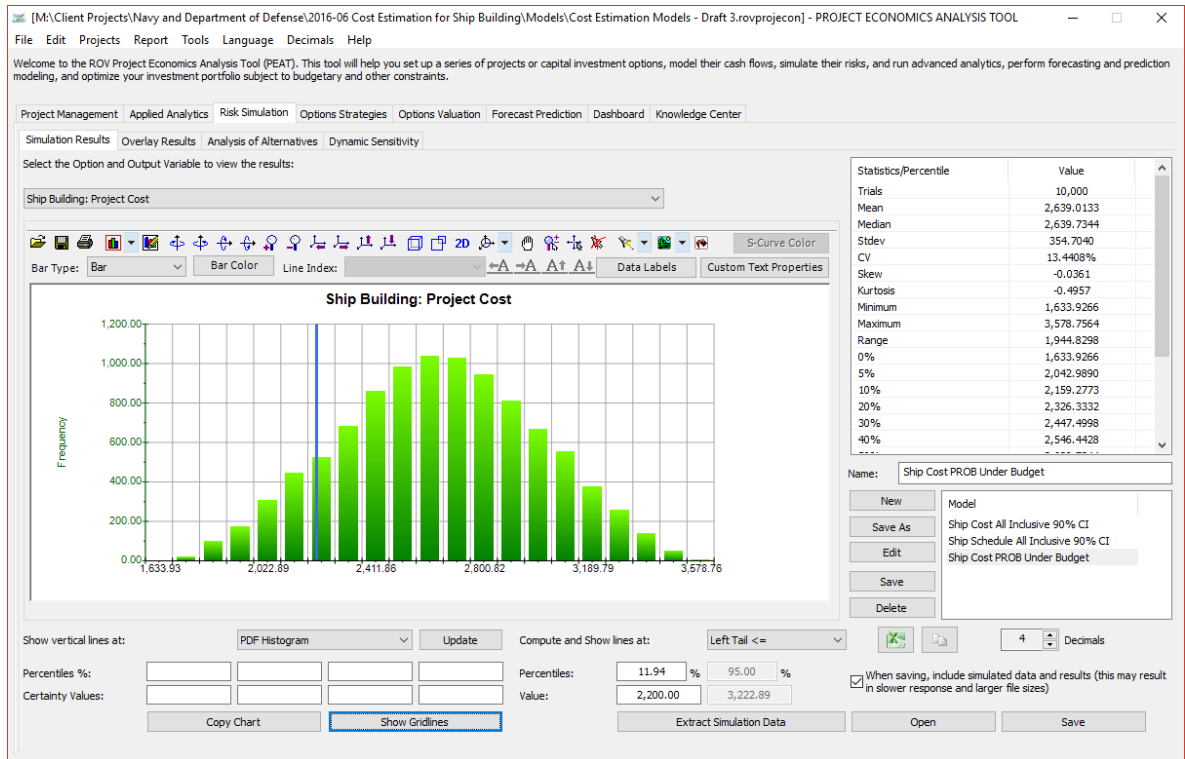


Figure 31: Probability of Cost Exceeding Budget

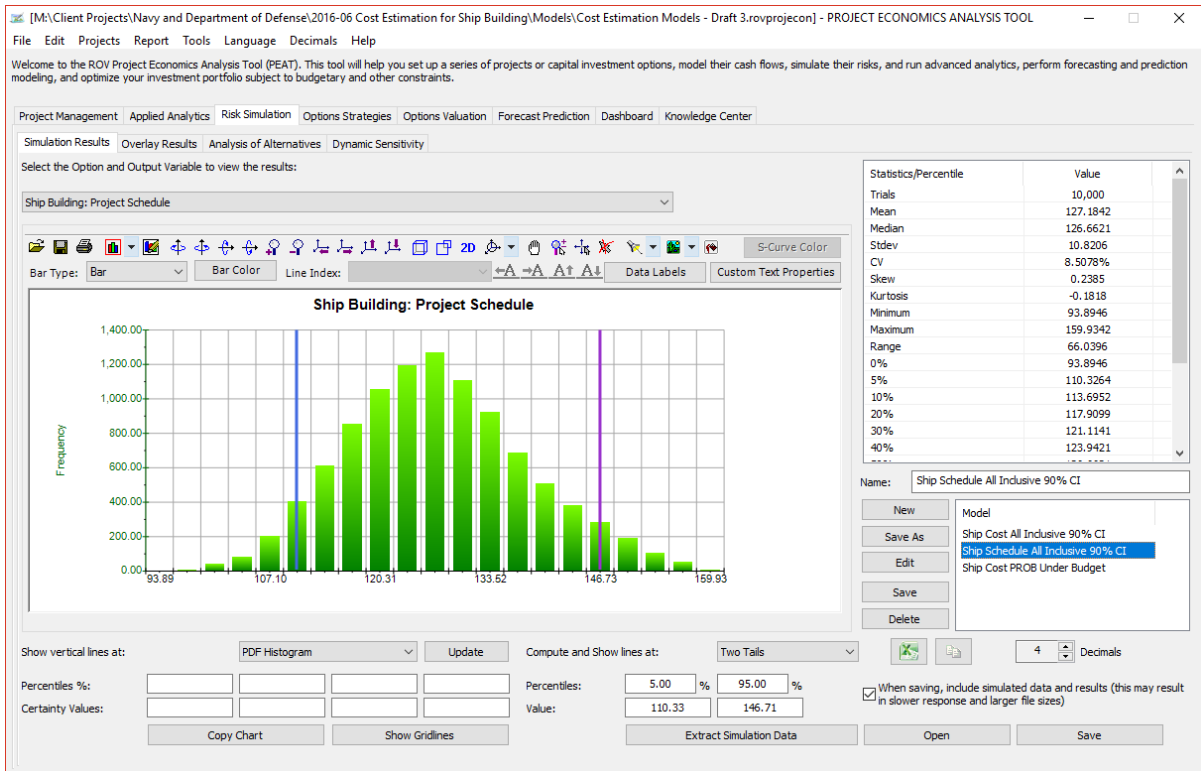


Figure 32: Schedule Risk (90% Confidence Interval)



Alternatively, the modeling approach allows us to look at the ship's subsystems. For example, Figure 33 shows the 90% confidence interval for weapons systems costs (\$1.1 to \$1.8 billion) or modeling the cost of building the ship without any subsystems (Figure 34). Each individual system or combinations of systems can be similarly modeled and analyzed (Figure 35), or overlaid on one another, as shown in Figures 36, 37, and 38. The probability distributions in these three figures allow you to compare how one system's cost and uncertainties around its costs compare to one another. Finally, Figure 39 shows how the individual tasks' schedule and cost elements impact and are correlated to each other, by way of dynamic sensitivity analysis.

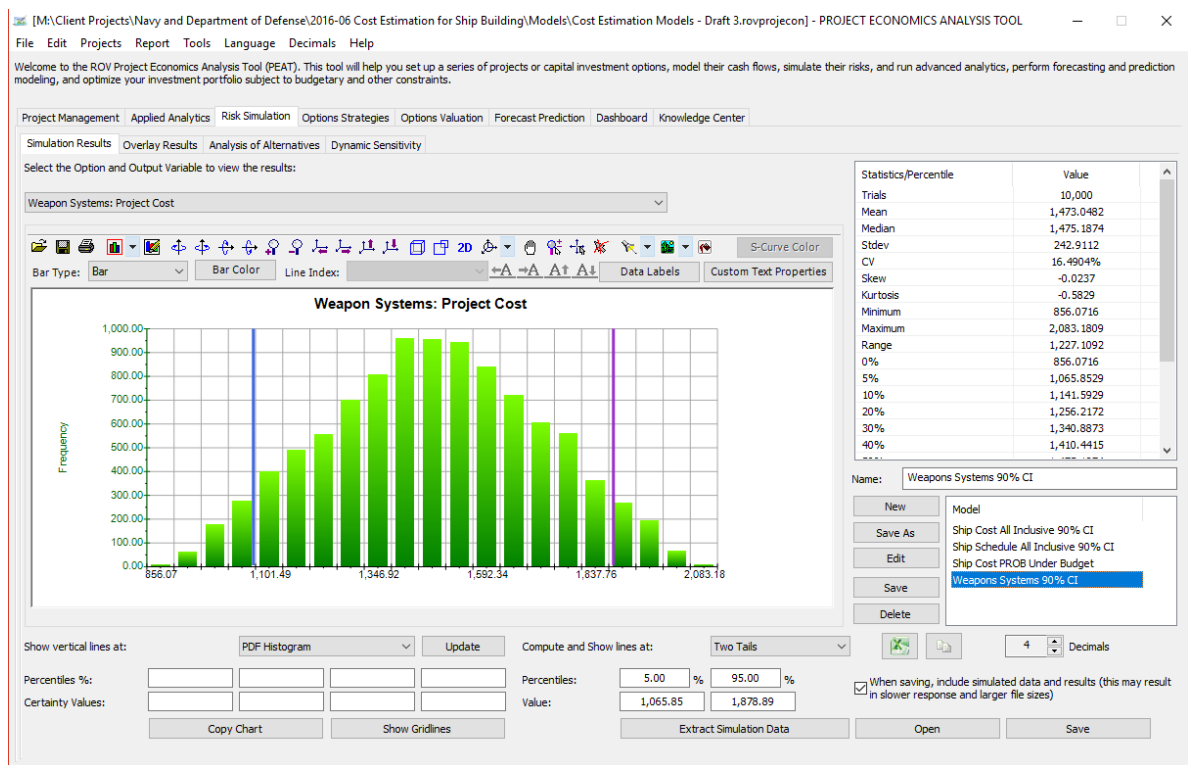


Figure 33: Cost of Weapons Systems (90% Confidence Interval)



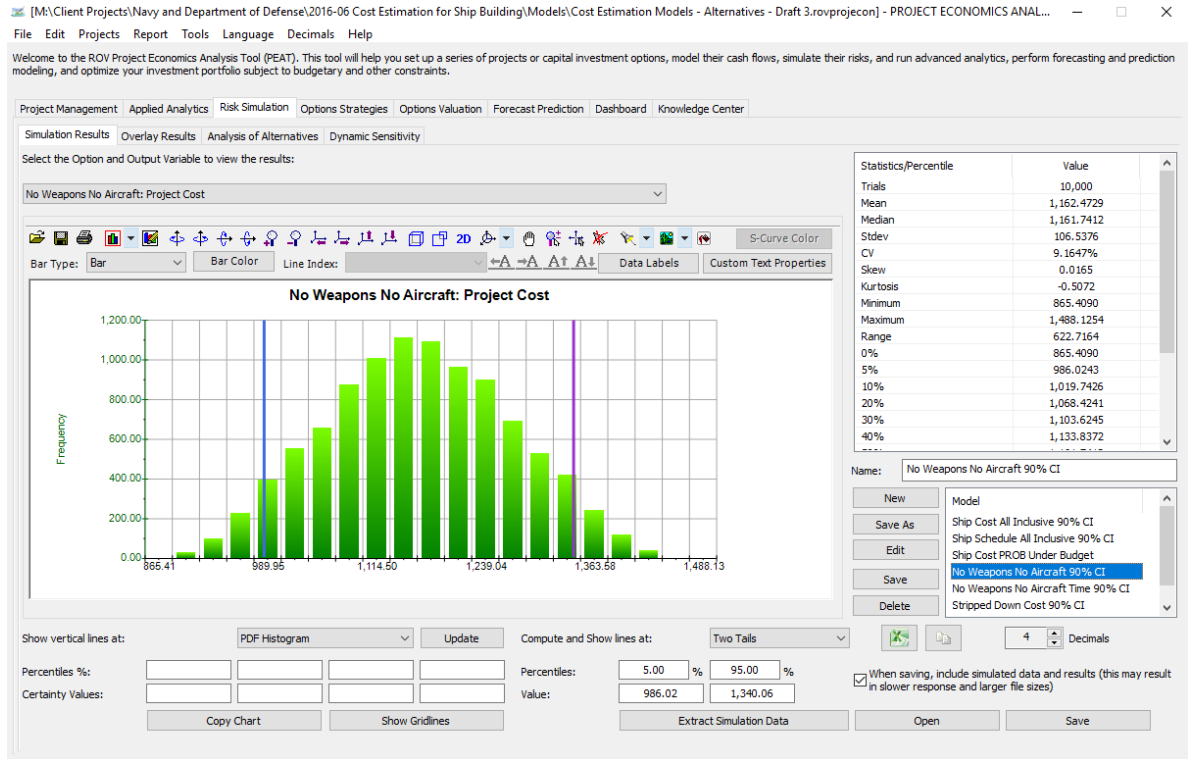


Figure 34: Simulated Cost of No Weapons and No Aircraft

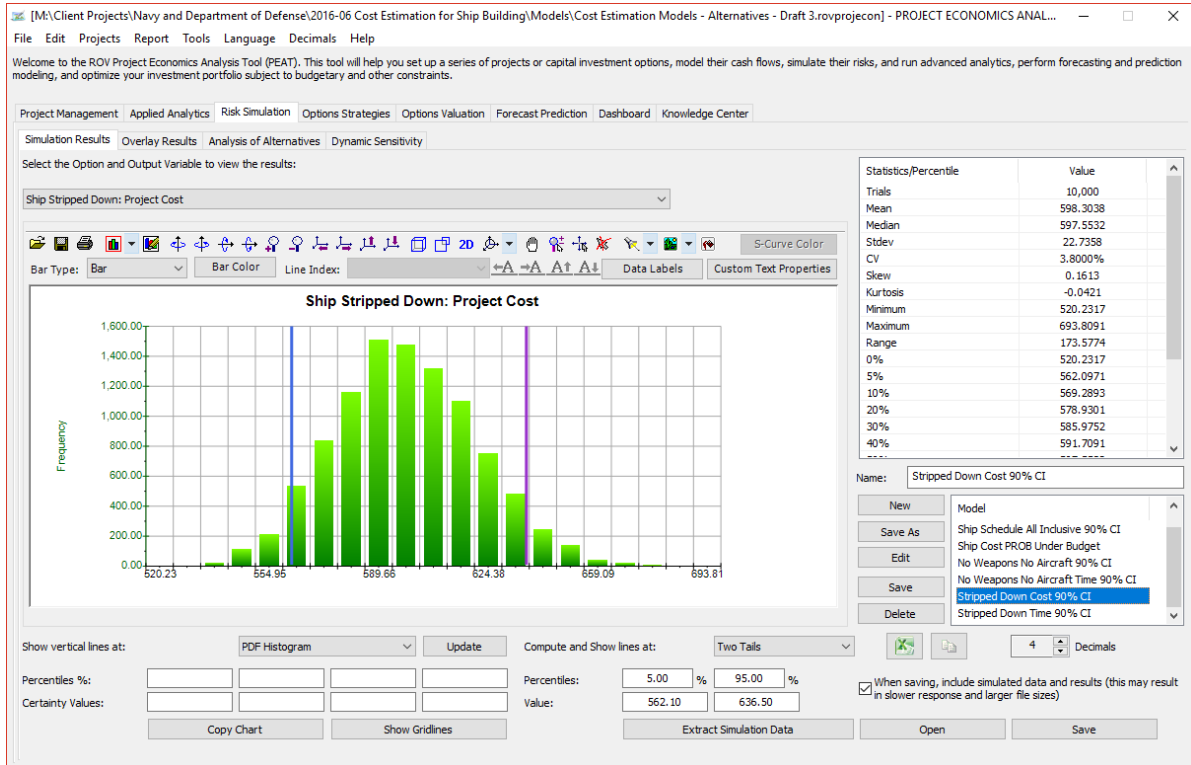


Figure 35: Simulated Cost of Stripped-Down Ship Build



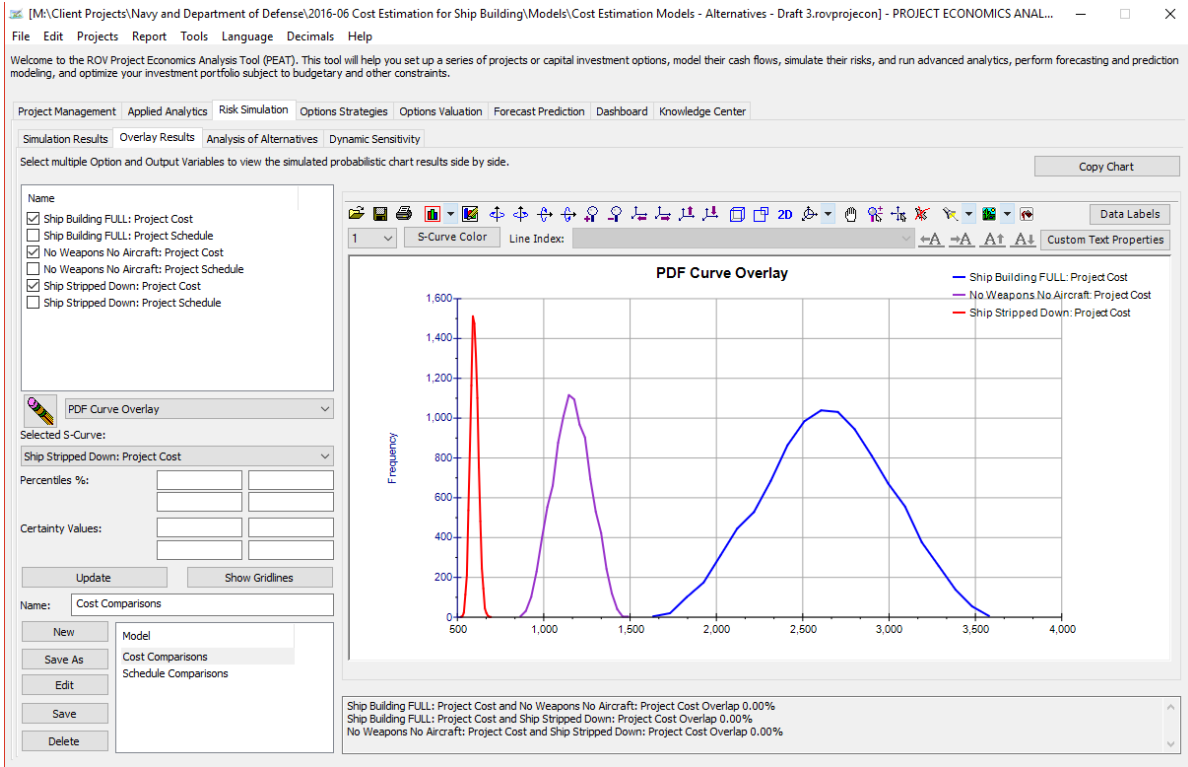


Figure 36: Comparative Analysis of Ship Configurations

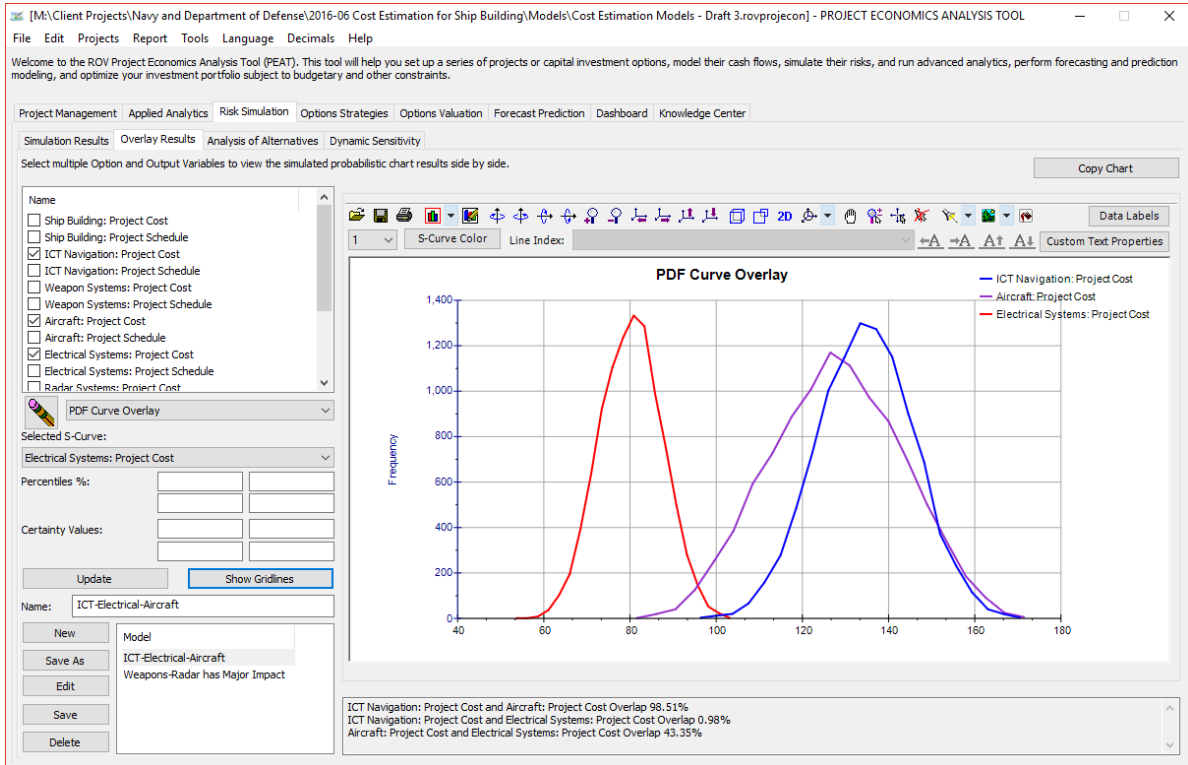


Figure 37: Overlay of Simulated Probability Distributions (Subsystems)



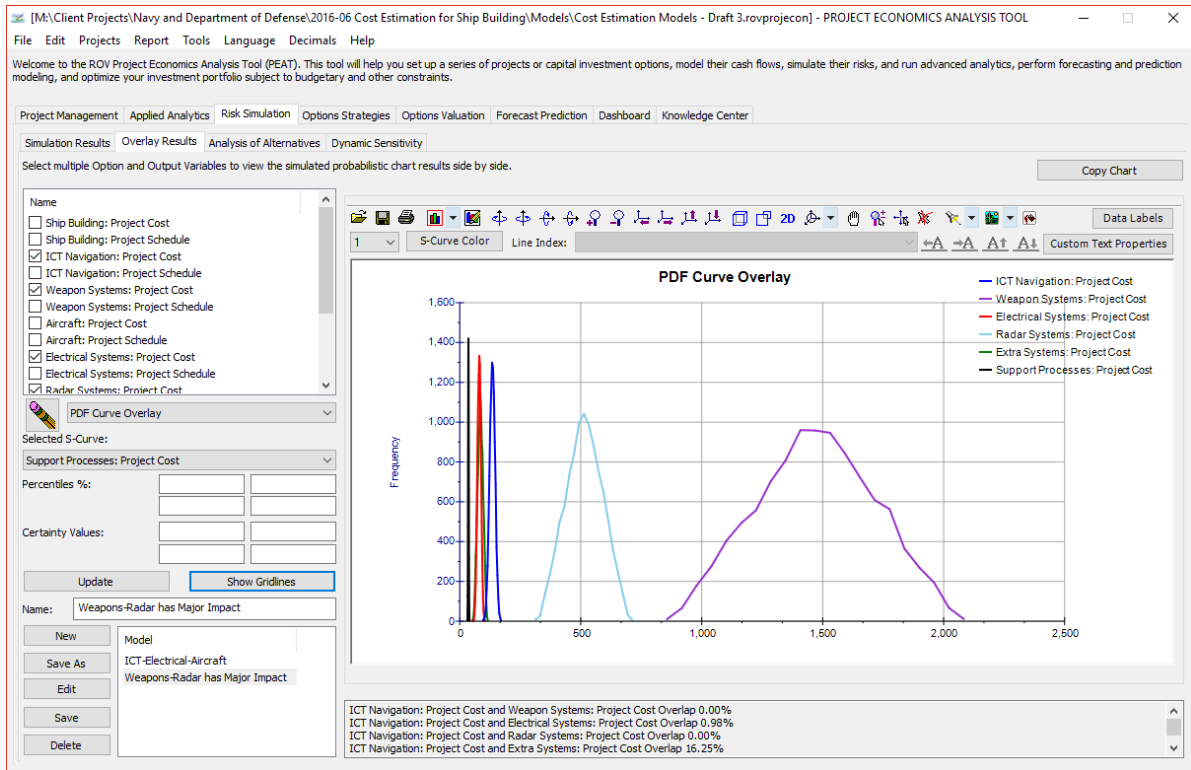


Figure 38: Overlay of Simulated Probability Distributions (All Subsystems)

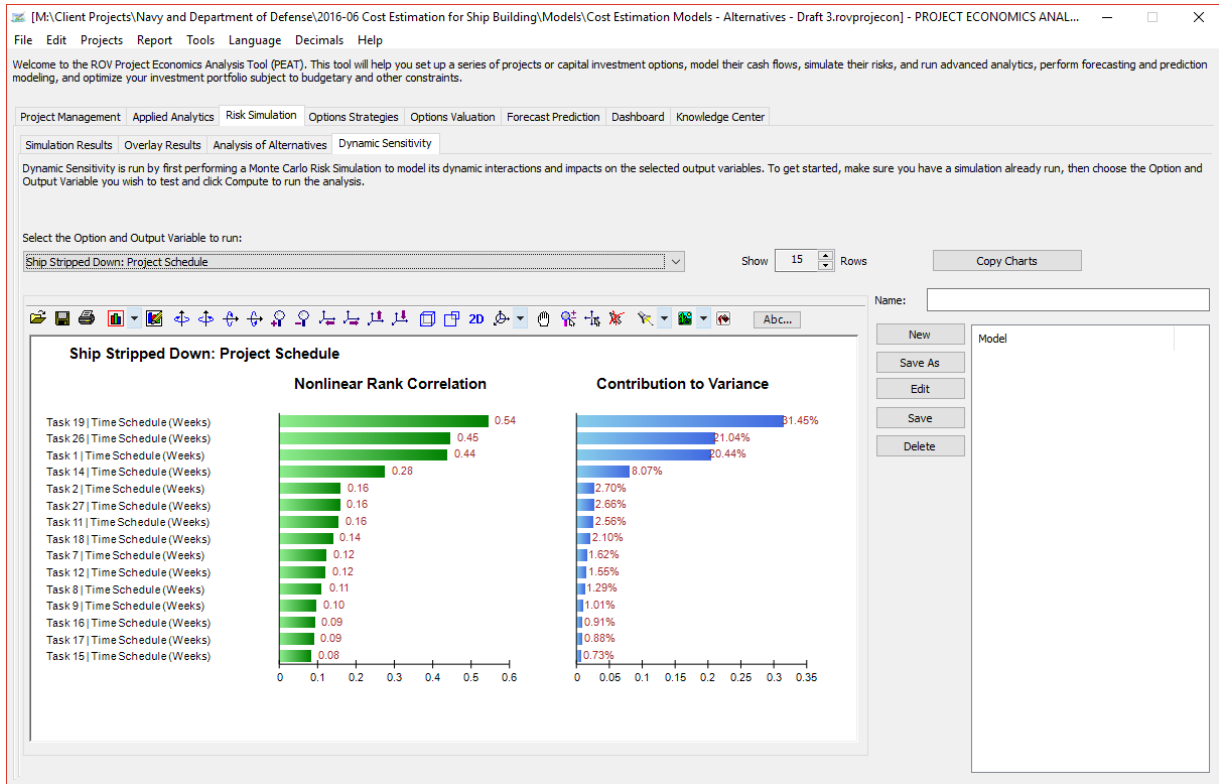


Figure 39: Dynamic Sensitivities of Stripped-Down Ship Build



Parametric Cost Models With Historical Data

A complementary approach to generate additional input cost assumptions includes the use of parametric modeling. In order to run parametric models (see *The Theory of Predictive Modeling in Cost* section), historical data is first required. Figure 40 shows an example dataset obtained via various defense agencies' publicly available information. The dataset shows various ship types, the unit costs (in millions), displacement in tons, speed, length, crew size, and year the ships were delivered.

Parametric models were developed and tested using simple multiple regression analysis, nonlinear regression, and econometric models. For instance, Figure 41 shows a simple linear parametric regression model and its results, where the functional form tested was:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$$

$$\text{Cost} = -11837 - 0.10 \text{ Tons} + 80.44 \text{ Speed} + 55.56 \text{ Length} + 6.09 \text{ Crew}$$



Multivariate Analysis (Warship Prices)

ID	Navy Ship	Unit Cost (\$M)	Displacement (Tons)	Speed (KMH)	Length (M)	Crew	Year	Value	Q
1	DDG 51	2133	9648	56	155.3	276	2012	2,133	1
2	DDG 51	1553	9648	56	155.3	276	2012	3,106	2
3	DDG 51	1884	9648	56	155.3	276	2012	1,884	1
4	DDG 51	1423	9648	56	155.3	276	2013	4,269	3
5	DDG 51	2372	9648	56	155.3	276	2014	2372	1
6	DDG 51	1615	9648	56	155.3	276	2015	1,615	1
7	DDG 51	1330.5	9648	56	155.3	276	2016	2,661	2
8	DDG 1000	3554	15730	56	185.9	148	2007	3554	1
9	DDG 1000	3010	15730	56	185.9	148	2008	3010	1
10	Joint High Speed Vessel (JHSV)	185	2397	80	103	41	2010	185	1
11	Joint High Speed Vessel (JHSV)	184	2397	80	103	41	2011	184	1
12	Joint High Speed Vessel (JHSV)	376	2397	80	103	41	2012	376	1
13	Joint High Speed Vessel (JHSV)	207	2397	80	103	41	2013	207	1
14	LHA 6 America	3204	45695	37	114.91	1,687	2007	3,204	1
15	LHA 6 America	3213	45695	37	114.91	1,687	2011	3,213	1
16	Littoral Combat Ship	1077	3292	87	115.3	45	2010	1,077	1
17	Littoral Combat Ship	1147	3293	87	115.3	45	2011	1,147	1
18	Littoral Combat Ship	1858	3294	87	115.3	45	2012	1,858	1
19	Littoral Combat Ship	1821	3295	87	115.3	45	2013	1,821	1
20	LPD 17 San Antonio Class	1903	25300	39	208.5	360	2009	1,903	1
21	LPD 17 San Antonio Class	2088	25300	39	208.5	360	2012	2,088	1
22	USS Ticonderoga (CG 47)	1000	9754	56	173	30	2008	1,000	1
23	DD-21 Zumwalt	2700	16000	56	170	150	1996	2,700	1
24	Nimitz Class Aircraft Carrier (CVN 68)	4045	99800	56	332.8	558	2009	4,045	1
25	Nimitz Class Aircraft Carrier (CVN 68)	3421.3	99800	56	332.8	558	2011	3,421	1
26	Nimitz Class Aircraft Carrier (CVN 68)	4568.8	99800	56	332.8	558	2012	4,569	1
27	Nimitz Class Aircraft Carrier (CVN 68)	4738.2	99800	56	332.8	558	2016	4,738	1

Similar methodology in "Why Has the Cost of Navy Ships Risen?" RAND National Defense Research Institute 2006
 Data Source: <http://www.bga-aeroweb.com/Defense/DDG-51-AEGIS-Destroyer.html>
<http://www.globalsecurity.org/military/systems/ship/ddg-51.htm>
<http://www.defenseindustrydaily.com/adding-arleigh-burkes-northrop-grumman-underway-06007/>

Figure 40: Sample Dataset for Parametric Modeling

Although the model looks good, with statistically significant *p*-values (e.g., 0.0097) that are lower than the standard 0.05 or 0.10 significance cutoffs, and the coefficients of determination (R-squared) are relatively high at 82.60%, the model is flawed. For instance, the coefficient for displacement is negative, which defies conventional logic, where typically the heavier the ship, the higher the cost. This means the model’s specification is incorrect and another model is required.



Figure 42 shows a mixed nonlinear parametric model with the following specification:

$$y = \beta_0 + \beta_1 \ln(X_1) + \beta_2 \ln(X_2) + \beta_3 X_3 + \beta_4 X_4$$

$$Cost = -40271 + 3351 \ln(Tons) + 3952 \ln(Speed) - 26.37 Length - 2.18 Crew$$

This model makes slightly more sense in that tonnage and speed have a positive relationship to cost and their effects are nonlinear. However, some of the other independent variables such as crew and length still show negative effects, albeit all modeled variables have the statistical significance of low p -values and a higher adjusted R-squared coefficient.

Regression Analysis Report

Regression Statistics

R-Squared (Coefficient of Determination)	0.8260
Adjusted R-Squared	0.7943
Multiple R (Multiple Correlation Coefficient)	0.9088
Standard Error of the Estimates (SEy)	585.1570
Number of Observations	27

The R-Squared or Coefficient of Determination indicates that 0.83 of the variation in the dependent variable can be explained and accounted for by the independent variables in this regression analysis. However, in a multiple regression, the Adjusted R-Squared takes into account the existence of additional independent variables or regressors and adjusts this R-Squared value to a more accurate view of the regression's explanatory power. Hence, only 0.79 of the variation in the dependent variable can be explained by the regressors.

The Multiple Correlation Coefficient (Multiple R) measures the correlation between the actual dependent variable (Y) and the estimated or fitted (Y) based on the regression equation. This is also the square root of the Coefficient of Determination (R-Squared).

The Standard Error of the Estimates (SEy) describes the dispersion of data points above and below the regression line or plane. This value is used as part of the calculation to obtain the confidence interval of the estimates later.

Regression Results

	Displacement				
	Intercept	(Tons)	Speed (KMH)	Length (M)	Crew
Coefficients	-11837.1869	-0.1034	80.4366	55.5622	6.0975
Standard Error	4077.1440	0.0365	29.5533	15.4242	1.7271
t-Statistic	-2.9033	-2.8328	2.7217	3.6023	3.5306
p-Value	0.0082	0.0097	0.0125	0.0016	0.0019
Lower 5%	-20292.6660	-0.1791	19.1467	23.5743	2.5158
Upper 95%	-3381.7078	-0.0277	141.7265	87.5501	9.6793

Degrees of Freedom

Degrees of Freedom for Regression	4
Degrees of Freedom for Residual	22
Total Degrees of Freedom	26

Hypothesis Test

Critical t-Statistic (99% confidence with df of 22)	2.8188
Critical t-Statistic (95% confidence with df of 22)	2.0739
Critical t-Statistic (90% confidence with df of 22)	1.7171

The Coefficients provide the estimated regression intercept and slopes. For instance, the coefficients are estimates of the true; population b values in the following regression equation $Y = b_0 + b_1X_1 + b_2X_2 + \dots + b_nX_n$. The Standard Error measures how accurate the predicted Coefficients are, and the t-Statistics are the ratios of each predicted Coefficient to its Standard Error.

The t-Statistic is used in hypothesis testing, where we set the null hypothesis (H_0) such that the real mean of the Coefficient = 0, and the alternate hypothesis (H_a) such that the real mean of the Coefficient is not equal to 0. A t-test is performed and the calculated t-Statistic is compared to the critical values at the relevant Degrees of Freedom for Residual. The t-test is very important as it calculates if each of the coefficients is statistically significant in the presence of the other regressors. This means that the t-test statistically verifies whether a regressor or independent variable should remain in the regression or it should be dropped.

The Coefficient is statistically significant if its calculated t-Statistic exceeds the Critical t-Statistic at the relevant degrees of freedom (df). The three main confidence levels used to test for significance are 90%, 95% and 99%. If a Coefficient's t-Statistic exceeds the Critical level, it is considered statistically significant. Alternatively, the p-Value calculates each t-Statistic's probability of occurrence, which means that the smaller the p-Value, the more significant the Coefficient. The usual significant levels for the p-Value are 0.01, 0.05, and 0.10, corresponding to the 99%, 95%, and 90% confidence levels.

The Coefficients with their p-Values highlighted in blue indicate that they are statistically significant at the 90% confidence or 0.10 alpha level, while those highlighted in red indicate that they are not statistically significant at any other alpha levels.

Figure 41: Simple Parametric Model with Linear Regression



Basic Econometrics Analysis Report

Regression Statistics	
R-Squared (Coefficient of Determination)	0.9027
Adjusted R-Squared	0.8850
Multiple R (Multiple Correlation Coefficient)	0.9501
Standard Error of the Estimates (SEy)	437.4818
Number of Observations	27

The R-Squared or Coefficient of Determination indicates that 0.90 of the variation in the dependent variable can be explained and accounted for by the independent variables in this regression analysis. However, in a multiple regression, the Adjusted R-Squared takes into account the existence of additional independent variables or regressors and adjusts this R-Squared value to a more accurate view of the regression's explanatory power. Hence, only 0.89 of the variation in the dependent variable can be explained by the regressors.

The Multiple Correlation Coefficient (Multiple R) measures the correlation between the actual dependent variable (Y) and the estimated or fitted (Y) based on the regression equation. This is also the square root of the Coefficient of Determination (R-Squared).

The Standard Error of the Estimates (SEy) describes the dispersion of data points above and below the regression line or plane. This value is used as part of the calculation to obtain the confidence interval of the estimates later.

Regression Results					
	Intercept	LN(VAR2)	LN(VAR3)	VAR4	VAR5
Coefficients	-40271.8660	3351.7927	3952.3622	-26.3671	-2.1820
Standard Error	7260.7364	596.1151	828.0072	6.9614	0.6903
t-Statistic	-5.5465	5.6227	4.7733	-3.7876	-3.1609
p-Value	0.0000	0.0000	0.0001	0.0010	0.0045
Lower 5%	-55329.7116	2115.5257	2235.1804	-40.8042	-3.6136
Upper 95%	-25214.0205	4588.0597	5669.5439	-11.9300	-0.7504

Degrees of Freedom		Hypothesis Test	
Degrees of Freedom for Regression	4	Critical t-Statistic (99% confidence with df of 22)	2.8188
Degrees of Freedom for Residual	22	Critical t-Statistic (95% confidence with df of 22)	2.0739
Total Degrees of Freedom	26	Critical t-Statistic (90% confidence with df of 22)	1.7171

Figure 42: Parametric Model with Nonlinear Regression

The econometric-based parametric model shown in Figure 43 is the best model both in significance as well as logic. For instance, there are polynomial functions and first order versus second order interactions of the independent variables. Specifically, the functional form producing the best-fitting mixed nonlinear parametric cost model is:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 \ln(X_1) + \beta_5 \ln(X_2) + \beta_6 \ln(X_3)$$

$$\begin{aligned} \text{Cost} = & 86373 - 0.37 \text{ Tons} + 302.18 \text{ Length} + 4.39 \text{ Crew} + 7108.91 \ln(\text{Tons}) \\ & + 9778.02 \ln(\text{Speed}) - 46327.8 \ln(\text{Length}) \end{aligned}$$

Clearly these are only illustrations based on sample publicly available data. Nonetheless, the approach is similar with actual data. The difference being only to use datasets that pertain to the ship that is being modeled to prevent out of sample biases. Additional independent variables will need to be collected, and various econometric tests will need to be performed (e.g., see Appendix 4 for an example list of specifications, data integrity, and error tests that will be performed, such as heteroskedasticity, multicollinearity, non-sphericity, nonlinearity, and so forth).



Auto Econometrics

Regression Statistics	
R-Squared (Coefficient of Determination)	0.9383
Adjusted R-Squared	0.9198
Multiple R (Multiple Correlation Coefficient)	0.9687
Standard Error of the Estimates (SEy)	365.4465
Number of Observations	27

The R-Squared or Coefficient of Determination indicates that 0.94 of the variation in the dependent variable can be explained and accounted for by the independent variables in this regression analysis. However, in a multiple regression, the Adjusted R-Squared takes into account the existence of additional independent variables or regressors and adjusts this R-Squared value to a more accurate view of the regression's explanatory power. Hence, only 0.92 of the variation in the dependent variable can be explained by the regressors.

The Multiple Correlation Coefficient (Multiple R) measures the correlation between the actual dependent variable (Y) and the estimated or fitted (Y) based on the regression equation. This is also the square root of the Coefficient of Determination (R-Squared).

The Standard Error of the Estimates (SEy) describes the dispersion of data points above and below the regression line or plane. This value is used as part of the calculation to obtain the confidence interval of the estimates later.

Regression Results							
	Intercept	var2	var4	var5	ln(var2)	ln(var3)	ln(var4)
Coefficients	86373.8318	-0.3741	302.1790	4.3956	7108.9055	9778.0160	-46327.8077
Standard Error	47165.1982	0.1184	108.1814	2.0715	1589.3175	1852.4014	16303.5560
t-Statistic	1.8313	-3.1603	2.7933	2.1220	4.4729	5.2786	-2.8416
p-Value	0.0820	0.0049	0.0112	0.0465	0.0002	0.0000	0.0101
Lower 5%	-12011.0457	-0.6211	76.5165	0.0746	3793.6472	5913.9744	-80336.4289
Upper 95%	184758.7092	-0.1272	527.8414	8.7166	10424.1637	13642.0575	-12319.1865

Degrees of Freedom		Hypothesis Test	
Degrees of Freedom for Regression	6	Critical t-Statistic (99% confidence with df of 20)	2.8453
Degrees of Freedom for Residual	20	Critical t-Statistic (95% confidence with df of 20)	2.0860
Total Degrees of Freedom	26	Critical t-Statistic (90% confidence with df of 20)	1.7247

Figure 43: Parametric Econometric Model with Nonlinear and Interacting Model

Parametric Probability Distribution and Curve Fitting

Another powerful cost modeling approach is distributional fitting; that is, how does an analyst or engineer determine which distribution to use for a particular task's input cost or schedule variable? What are the relevant distributional parameters? If no historical data exist, we can make assumptions about the variables in question using the Delphi method, where a group of subject matter experts are tasked with estimating the behavior of each variable. These values can be used as the variable's input parameters (e.g., uniform distribution with extreme values between 0.5 and 1.2). When testing is not possible (e.g., a new or novel weapon subsystem), management can still make estimates of potential outcomes and provide the best-case, most-likely case, and worst-case scenarios, whereupon a triangular or custom distribution can be created.

However, if reliable historical data are available, distributional fitting can be accomplished. Assuming that historical patterns hold and that history tends to repeat itself, then historical data can be used to find the best-fitting distribution with their



relevant parameters to better define the variables to be simulated. Figure 44 illustrate a distributional-fitting example of the costs shown previously (Figure 40).

The null hypothesis (H_0) being tested is such that the fitted distribution is the same distribution as the population from which the sample data to be fitted come. Thus, if the computed p -value is lower than a critical alpha level (typically .10 or .05), then the distribution is the wrong distribution. Conversely, the higher the p -value, the better the distribution fits the data. Roughly, you can think of p -value as a percentage explained, that is, if the p -value is 0.9849 (Figure 44), then setting a normal distribution with a mean of 1990 and a standard deviation of 1290 explains about 98.49% of the variation in the data, indicating an especially good fit. The results also rank all the selected distributions and how well they fit the data. The fitted distribution can now be set up to run a simulation. The results from the simulation (tens to hundreds of thousands of simulation trials can be run) can be interpreted as usual (Figure 45).

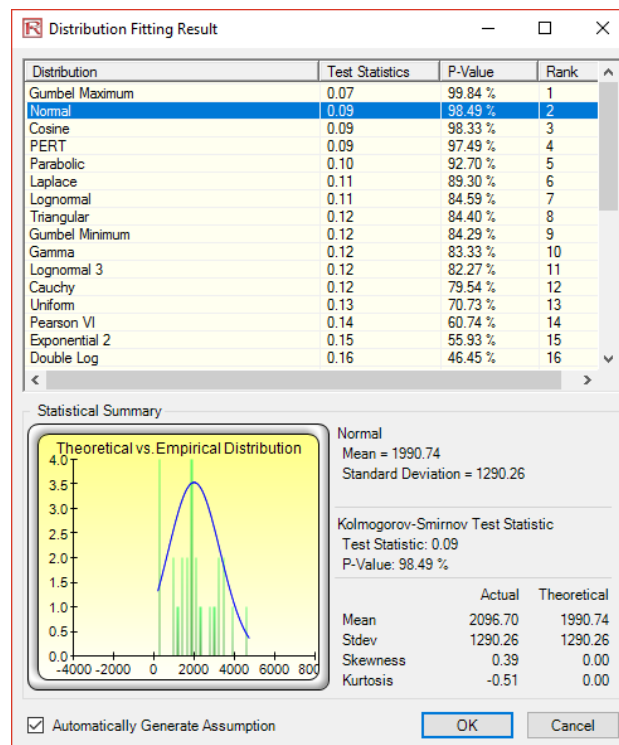


Figure 44: Parametric Monte Carlo Simulation Model Distributional Fitting

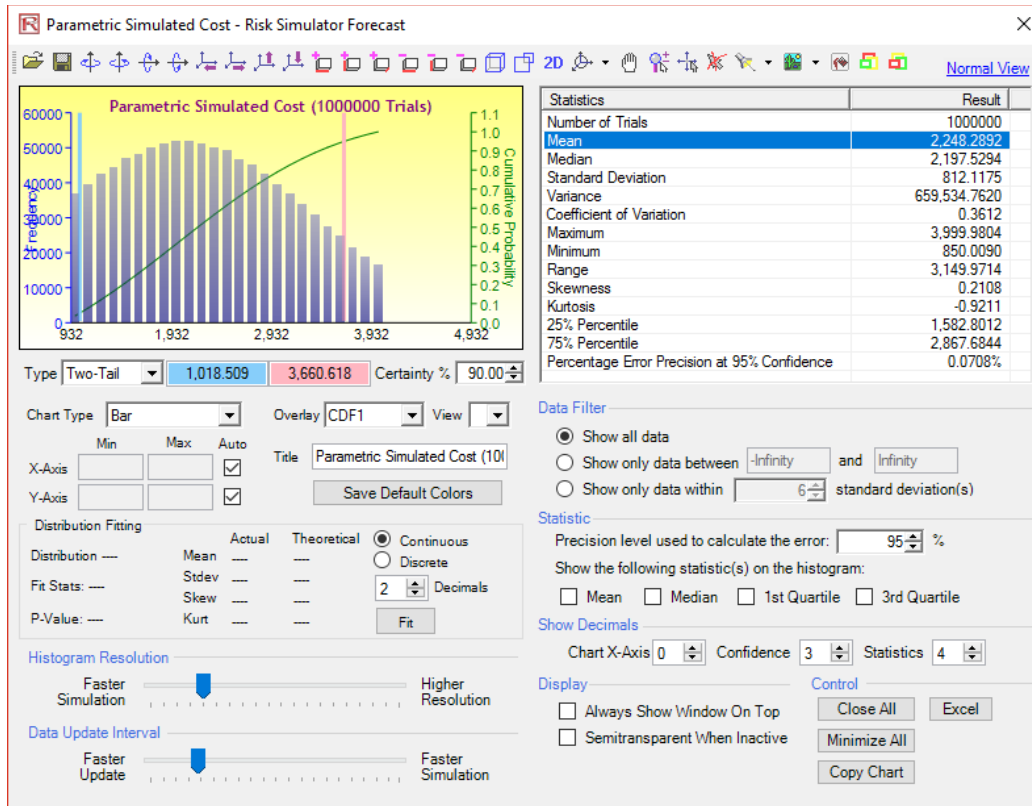


Figure 45: Parametric Simulated Cost Results



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Conclusions and Next Step Recommendations

Based on this current preliminary analysis, we conclude that the risk-based cost and schedule simulations as well as parametric econometric models can be suitably applied to modeling the cost of current and future U.S. Navy warships. It is evident in the analysis that any cost modeling must also include schedule risk as schedule delays can cause significant cost creeps and budget overruns. Using the project process diagrams and task-based cost modeling, coupled with Monte Carlo simulations to account for uncertainties in input assumptions and estimates and risks of overruns, a more comprehensive methodology was developed.

We therefore recommend the following:

- Collecting and using actual cost data and better cost estimates going forward in order to better calibrate the inputs based on real-life conditions. (We can provide inputs and suggestions on how to generate a database and methods to capture said required data.)
- Using the simulated probability distributions to determine how well the vendors are performing (e.g., running at 92% efficiency, etc.), thus creating a level of performance metrics for the organization.
- Using control charts (based on simulated results) to determine if any processes and tasks are in-control or out-of-control over time.
- Identifying critical success factors to start collecting cost and schedule data for better estimates.
- Incorporating learning curves and synergies when more than one ship is in order and the unit cost per ship would be lower.



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Appendix 1—A Primer on Integrated Risk Management

Since the beginning of recorded history, games of chance have been a popular pastime. Even in Biblical accounts, Roman soldiers cast lots for Christ's robes. In earlier times, chance was something that occurred in nature, and humans were simply subjected to it as a ship is to the capricious tosses of the waves in an ocean. Even up to the time of the Renaissance, the future was thought to be simply a chance occurrence of completely random events and beyond the control of humans. However, with the advent of games of chance, human greed has propelled the study of risk and chance to ever more closely mirror real-life events. Although these games were initially played with great enthusiasm, no one actually sat down and figured out the odds. Of course, the individual who understood and mastered the concept of chance was bound to be in a better position to profit from such games of chance. It was not until the mid-1600s that the concept of chance was properly studied, and the first such serious endeavor can be credited to Blaise Pascal, one of the fathers of the study of choice, chance, and probability. Fortunately for us, after many centuries of mathematical and statistical innovations from pioneers such as Pascal, Bernoulli, Bayes, Gauss, LaPlace, and Fermat, and with the advent of blazing fast computing technology, our modern world of uncertainty can be explained with much more elegance through methodological rigorous hands-on applications of risk and uncertainty. Even as recent as two and a half decades ago, computing technology was only in its infancy and running complex and advanced analytical models would have seemed a fantasy, but today, with the assistance of more powerful and enabling software packages, we have the ability to practically apply such techniques with great ease. For this reason, we have chosen to learn from human history that with innovation comes the requisite change in human behavior to apply these new methodologies as the new norm for rigorous risk-benefit analysis.

To the people who lived centuries ago, risk was simply the inevitability of chance occurrence beyond the realm of human control. Albeit many phony



soothsayers profited from their ability to convincingly profess their clairvoyance by simply stating the obvious or reading the victims' body language and telling them what they wanted to hear. We modern-day humans, ignoring for the moment the occasional seers among us, with our fancy technological achievements, are still susceptible to risk and uncertainty. We may be able to predict the orbital paths of planets in our solar system with astounding accuracy, or to predict the escape velocity required to shoot a man from the Earth to the Moon, or to drop a smart bomb within a few feet of its target thousands of miles away, but when it comes to, say, predicting a firm's revenues the following year, we are at a loss. Humans have been struggling with risk our entire existence, but through trial and error, and through the evolution of human knowledge and thought, we have devised ways to describe, quantify, hedge, and take advantage of risk.

In the U.S. military context, risk analysis, real options analysis, and portfolio optimization techniques are enablers of a new way of approaching the problems of estimating return on investment (ROI) and estimating the risk-value of various strategic real options. There are many new Department of Defense (DoD) requirements for using more advanced analytical techniques. For instance, the Clinger-Cohen Act of 1996 mandates the use of portfolio management for all federal agencies. The Government Accountability Office's (1997) *Assessing Risks and Returns: A Guide for Evaluating Federal Agencies' IT Investment Decision-Making*, requires that IT investments apply ROI measures. DoD Directive 8115.01, issued in October 2005, mandates the use of performance metrics based on outputs, with ROI analysis required for all current and planned IT investments. DoD Directive 8115.bb implements policy and assigns responsibilities for the management of DoD IT investments as portfolios within the DoD Enterprise where they defined a portfolio to include outcome performance measures and an expected return on investment. The DoD Risk Management Guidance Defense Acquisition guide book requires that alternatives to the traditional cost estimation need to be considered because legacy cost models tend not to adequately address costs associated with information systems or the risks associated with them.



In this quick primer, advanced quantitative risk-based concepts will be introduced, namely, the hands-on applications of Monte Carlo simulation, real options analysis, stochastic forecasting, portfolio optimization, and knowledge value added. These methodologies rely on common metrics and existing techniques (e.g., return on investment, discounted cash flow, cost-based analysis, and so forth), and complement these traditional techniques by pushing the envelope of analytics, and not replacing them outright. It is not a complete change of paradigm, and we are not asking the reader to throw out what has been tried and true, but to shift one's paradigm, to move with the times, and to improve upon what has been tried and true. These new methodologies are used in helping make the best possible decisions, allocate budgets, predict outcomes, create portfolios with the highest strategic value and returns on investment, and so forth, where the conditions surrounding these decisions are risky or uncertain. They can be used to identify, analyze, quantify, value, predict, hedge, mitigate, optimize, allocate, diversify, and manage risk for military options.

Why Is Risk Important in Making Decisions?

Before we embark on the journey to review these advanced techniques, let us first consider why risk is critical when making decisions, and how traditional analyses are inadequate in considering risk in an objective way. Risk is an important part of the decision-making process. For instance, suppose projects are chosen based simply on an evaluation of returns alone or cost alone; clearly the higher-return or lower-cost project will be chosen over lower-return or higher-cost projects.

As mentioned, projects with higher returns will in most cases bear higher risks. And those projects with immediately lower returns would be abandoned. In those cases, where return estimates are wholly derived from cost data (with some form of cost in the numerator and denominator of ROI), the best thing to do is reduce all the costs, that is, never invest in new projects. The result of this primary focus on cost reduction is a stifling of innovation and new ways of doing things. The goal is not simply cost reduction. In this case, the simplest approach is to fire everyone and



sell off all the assets. The real question that must be answered is how cost compares to desired outputs, that is, “cost compared to what?”

To encourage a focus on improving processes and innovative technologies, a new way of calculating return on investment that includes a unique numerator is required. ROI is a basic productivity ratio that requires unique estimates of the numerator (i.e., value, revenue in common units of measurement) and the denominator (i.e., costs, investments in dollars). ROI estimates must be placed within the context of a longer term view that includes estimates of risk and the ability of management to adapt as they observe the performance of their investments over time. Therefore, instead of relying purely on immediate ROIs or costs, a project, strategy, process innovation, or new technology should be evaluated based on its total strategic value, including returns, costs, and strategic options, as well as its risks. Figures A.1 and A.2 illustrate the errors in judgment when risks are ignored. Figure A.1 lists three mutually exclusive projects with their respective costs to implement, expected net returns (net of the costs to implement), and risk levels (all in present values). Clearly, for the budget-constrained decision maker, the cheaper the project the better, resulting in the selection of Project X. The returns-driven decision maker will choose Project Y with the highest returns, assuming that budget is not an issue. Project Z will be chosen by the risk-averse decision maker as it provides the least amount of risk while providing a positive net return. The upshot is that, with three different projects and three different decision makers, three different decisions will be made. Who is correct and why?



Why is Risk Important?			
Name of Project	Cost	Returns	Risk
Project X	\$50	\$50	\$25
Project Y	\$250	\$200	\$200
Project Z	\$100	\$100	\$10

Project X for the cost and budget-constrained manager
 Project Y for the returns driven and nonresource-constrained manager
 Project Z for the risk-adverse manager
 Project Z for the smart manager

Figure A.1. Why Is Risk Important?

Figure A.2 shows that Project Z should be chosen. For illustration purposes, suppose all three projects are independent and mutually exclusive, and that an unlimited number of projects from each category can be chosen but the budget is constrained at \$1,000. Therefore, with this \$1,000 budget, 20 project Xs can be chosen, yielding \$1,000 in net returns and \$500 risks, and so forth. It is clear from Figure A.2 that project Z is the best project as for the same level of net returns (\$1,000), the least amount of risk is undertaken (\$100). Another way of viewing this selection is that for each \$1 of returns obtained, only \$0.10 of risk is involved on average, or that for each \$1 of risk, \$10 in returns are obtained on average. This example illustrates the concept of bang for the buck or getting the best value (benefits and costs both considered) with the least amount of risk. An even more blatant example is if there are several different projects with identical single-point average net benefit or cost of \$10 million each. Without risk analysis, a decision maker should in theory be indifferent in choosing any of the projects. However, with risk analysis, a better decision can be made. For instance, suppose the first project has a 10% chance of exceeding \$10 million, the second a 15% chance, and the third a 55% chance. Additional critical information is obtained on the riskiness of the project or strategy and a better decision can be made.



Adding an Element of Risk...

Looking at bang for the buck, X (2), Y (1), Z (10), Project Z should be chosen—with a \$1,000 budget, the following can be obtained:

Project X: 20 Project Xs returning \$1,000, with \$500 risk
Project Y: 4 Project Xs returning \$800, with \$800 risk
Project Z: 10 Project Xs returning \$1,000, with \$100 risk

Project X: For each \$1 return, \$0.5 risk is taken
Project Y: For each \$1 return, \$1.0 risk is taken
Project Z: For each \$1 return, \$0.1 risk is taken

Project X: For each \$1 of risk taken, \$2 return is obtained
Project Y: For each \$1 of risk taken, \$1 return is obtained
Project Z: For each \$1 of risk taken, \$10 return is obtained

Conclusion:

Risk is important. Foregoing risks results in making the wrong decision.

Figure A.2. Adding an Element of Risk

From Dealing With Risk the Traditional Way to Monte Carlo Simulation

Military and business leaders have been dealing with risk since the beginning of the history of war and commerce. In most cases, decision makers have looked at the risks of a particular project, acknowledged their existence, and moved on. Little quantification was performed in the past. In fact, most decision makers look only to single-point estimates of a project's benefit or profitability. Figure A.3 shows an example of a single-point estimate. The estimated net revenue of \$30 is simply that, a single point whose probability of occurrence is close to zero. Even in the simple model shown in Figure A.3, the effects of interdependencies are ignored, and in traditional modeling jargon, we have the problem of garbage-in, garbage-out (GIGO). As an example of interdependencies, the units sold are probably negatively correlated to the price of the product, and positively correlated to the average variable cost; ignoring these effects in a single-point estimate will yield grossly incorrect results. There are numerous interdependencies in military options as well, for example, the many issues in logistics and troop movements beginning with the manufacturer all the way to the warrior in the field.



In the commercial example below, if the unit sales variable becomes 11 instead of 10, the resulting revenue may not simply be \$35. The net revenue may actually decrease due to an increase in variable cost per unit while the sale price may actually be slightly lower to accommodate this increase in unit sales. Ignoring these interdependencies will reduce the accuracy of the model.

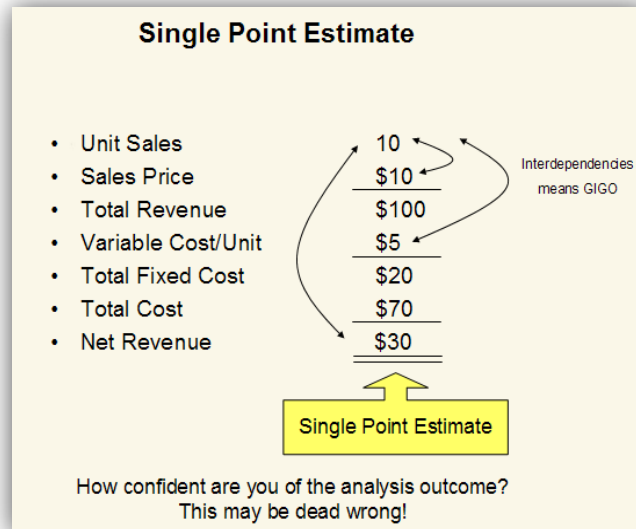


Figure A.3. Single-Point Estimates

One traditional approach used to deal with risk and uncertainty is the application of scenario analysis. For example, scenario analysis is a central part of the capabilities-based planning approach in widespread use for developing DoD strategies. In the commercial example above, suppose three scenarios were generated: the worst-case, nominal-case, and best-case scenarios. When different values are applied to the unit sales, the resulting three scenarios' net revenues are obtained. As earlier, the problems of interdependencies are not addressed with these common approaches. The net revenues obtained are simply too variable. Not much can be determined from such an analysis.

In the military planning case, the problems are exacerbated by the lack of objective ways to estimate benefits in common units. Without the common-unit benefits analysis, it becomes difficult, if not impossible, to compare the net benefits of various scenarios. In addition, interdependencies must be interpreted in a largely

subjective manner, making it impossible to apply powerful mathematical and statistical tools that enable more objective portfolio analysis. The problem arises for the top leaders in the DoD to make judgment calls, or selections among alternatives (often referred to as “trades”) about the potential benefits and risks of numerous projects and technologies investments.

A related approach is to perform a what-if or sensitivity analysis. Each variable is perturbed a prespecified amount (e.g., unit sales is changed $\pm 10\%$, sales price is changed $\pm 5\%$, and so forth) and the resulting change in net benefits is captured. This approach is useful for understanding which variables drive or impact the result the most. Performing such analyses by hand or with simple Excel spreadsheets is tedious and provides marginal benefits at best. A related approach that has the same goals but employs a more powerful analytic framework is the use of computer-modeled Monte Carlo simulation and tornado sensitivity analysis, where all perturbations, scenarios, and sensitivities are run hundreds of thousands of times automatically.

Therefore, computer-based Monte Carlo simulation, one of the advanced concepts introduced in this paper, can be viewed as simply an extension of the traditional approaches of sensitivity and scenario testing. The critical success drivers or the variables that affect the bottom-line variables the most, which at the same time are uncertain, are simulated. In simulation, the interdependencies are accounted for by using correlation analysis. The uncertain variables are then simulated tens of thousands of times automatically to emulate all potential permutations and combinations of outcomes. The resulting net revenues-benefits from these simulated potential outcomes are tabulated and analyzed. In essence, in its most basic form, simulation is simply an enhanced version of traditional approaches, such as sensitivity and scenario analysis, but automatically performed for thousands of times while accounting for all the dynamic interactions between the simulated variables. The resulting net revenues from simulation, as seen in Figure A.4, show that there is a 90% probability that the net revenues will fall between \$19.44 and \$41.25, with a 5% worst-case scenario of net revenues falling below \$19.44. Rather than having only three scenarios, simulation created 5,000



scenarios, or trials, where multiple variables are simulated and changing simultaneously (unit sales, sale price, and variable cost per unit), while their respective relationships or correlations are maintained.

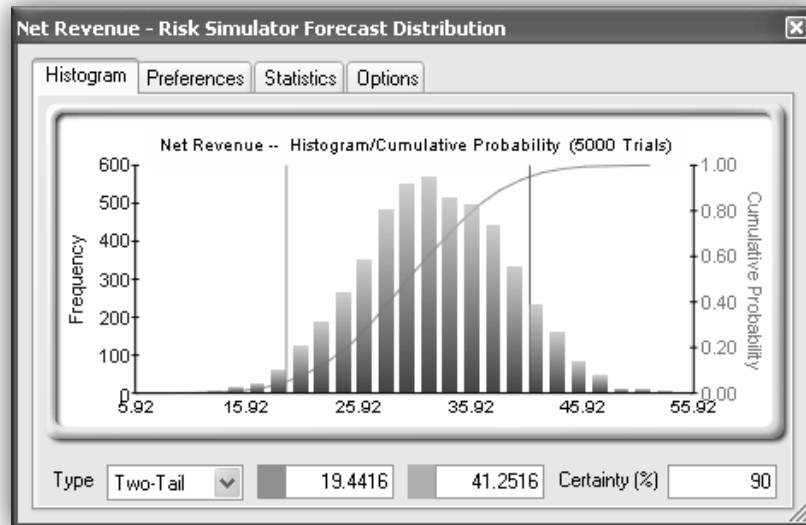


Figure A.4. Simulation Results

Monte Carlo simulation, named for the famous gambling capital of Monaco, is a very potent methodology. For the practitioner, simulation opens the door for solving difficult and complex but practical problems with great ease. Perhaps the most famous early use of Monte Carlo simulation was by the Nobel physicist Enrico Fermi (sometimes referred to as the father of the atomic bomb) in 1930, when he used a random method to calculate the properties of the newly discovered neutron. Monte Carlo methods were central to the simulations required for the Manhattan Project, where, in the 1950s, Monte Carlo simulation was used at Los Alamos for early work relating to the development of the hydrogen bomb and became popularized in the fields of physics and operations research. The Rand Corporation and the U.S. Air Force were two of the major organizations responsible for funding and disseminating information on Monte Carlo methods during this time, and today there is a wide application of Monte Carlo simulation in many different fields including engineering, physics, research and development, business, and finance.

Simplistically, Monte Carlo simulation creates artificial futures by generating thousands and even hundreds of thousands of sample paths of outcomes and analyzes their prevalent characteristics. In practice, Monte Carlo simulation methods are used for risk analysis, risk quantification, sensitivity analysis, and prediction. An alternative to simulation is the use of highly complex stochastic closed-form mathematical models. For a high-level decision maker, taking graduate level advanced math and statistics courses is just not logical or practical. A well-informed analyst would use all available tools at his or her disposal to obtain the same answer the easiest and most practical way possible. And in all cases, when modeled correctly, Monte Carlo simulation provides similar answers to the more mathematically elegant methods. In addition, there are many real-life applications where closed-form models do not exist and the only recourse is to apply simulation methods. So, what exactly is Monte Carlo simulation and how does it work?

Monte Carlo simulation in its simplest form is a random number generator that is useful for forecasting, estimation, and risk analysis. A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined probability distribution for the uncertain variables and using those values for the model. As all those scenarios produce associated results in a model, each scenario can have a forecast. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model.

Think of the Monte Carlo simulation approach as picking golf balls out of a large basket repeatedly with replacement. The size and shape of the basket depend on the distributional input assumption (e.g., a normal distribution with a mean of 100 and a standard deviation of 10, versus a uniform distribution or a triangular distribution) where some baskets are deeper or more symmetrical than others, allowing certain balls to be pulled out more frequently than others. The number of balls pulled repeatedly depends on the number of trials simulated. Each ball is indicative of an event, scenario, or condition that can occur. For a large model with multiple related assumptions, imagine the large model as a very large basket, wherein many baby baskets reside. Each baby basket has its own set of colored golf balls that are bouncing around. Sometimes these baby baskets are linked with each



other (if there is a correlation between the variables), forcing the golf balls to bounce in tandem whereas in other uncorrelated cases, the balls are bouncing independently of one another. The balls that are picked each time from these interactions within the model (the large basket) are tabulated and recorded, providing a forecast output result of the simulation.

Knowledge Value Added Analysis

As the U.S. Military is not in the business of making money, referring to revenues throughout this paper may appear to be a misnomer. For nonprofit organizations, especially in the military, we require Knowledge Value Added (KVA), which will provide the required “benefits” or “revenue” proxy estimates to run ROI analysis. ROI is a basic productivity ratio with revenue in the numerator and cost to generate the revenue in the denominator (actually ROI is revenue-cost/cost). KVA generates ROI estimates by developing a market comparable price per common unit of output multiplied by the number of outputs to achieve a total revenue estimate.

KVA is a methodology whose primary purpose is to describe all organizational outputs in common units. It provides a means to compare the outputs of all assets (human, machine, information technology) regardless of the aggregated outputs produced. For example, the purpose of a military process may be to gather signal intelligence or plan for a ship alteration. KVA would describe the outputs of both processes in common units thus making their performance comparable.

KVA measures the value provided by human capital assets and IT assets by analyzing an organization, process, or function at the process level. It provides insights into each dollar of IT investment by monetizing the outputs of all assets, including intangible assets (e.g., assets produced by IT and humans). By capturing the value of knowledge embedded in an organization’s core processes (i.e., employees and IT), KVA identifies the actual cost and revenue of a process, product, or service. Because KVA identifies every process required to produce an aggregated output in terms of the historical prices and costs per common unit of output of those processes, unit costs and unit prices can be calculated. The



methodology has been applied in 45 areas within the DoD, from flight scheduling applications to ship maintenance and modernization processes.

As a performance tool, the KVA methodology

- compares all processes in terms of relative productivity,
- allocates revenues and costs to common units of output,
- measures value added by IT by the outputs it produces, and
- relates outputs to cost of producing those outputs in common units.

Based on the tenets of complexity theory, KVA assumes that humans and technology in organizations add value by taking inputs and changing them (measured in units of complexity) into outputs through core processes. The amount of change an asset within a process produces can be a measure of value or benefit. The additional assumptions in KVA include the following:

- Describing all process outputs in common units (e.g., using a knowledge metaphor for the descriptive language in terms of the time it takes an average employee to learn how to produce the outputs) allows historical revenue and cost data to be assigned to those processes historically.
- All outputs can be described in terms of the time required to learn how to produce them.
- Learning Time, a surrogate for procedural knowledge required to produce process outputs, is measured in common units of time. Consequently, Units of Learning Time = Common Units of Output (K).
- A common unit of output makes it possible to compare all outputs in terms of cost per unit as well as price per unit, because revenue can now be assigned at the suborganizational level.
- Once cost and revenue streams have been assigned to suborganizational outputs, normal accounting and financial performance and profitability metrics can be applied (Rodgers & Housel, 2006; Pavlou et al., 2005; Housel & Kanevsky, 1995).



Describing processes in common units also permits market comparable data to be generated, which is particularly important for nonprofits like the U.S. Military. Using a market comparable approach, data from the commercial sector can be used to estimate price per common unit, allowing for revenue estimates of process outputs for nonprofits. This approach also provides a common-unit basis to define benefit streams regardless of the process analyzed.

KVA differs from other nonprofit ROI models because it allows for revenue estimates, enabling the use of traditional accounting, financial performance, and profitability measures at the sub organizational level. KVA can rank processes by the degree to which they add value to the organization or its outputs. This ranking assists decision makers to identify how much processes add value. Value is quantified in two key metrics: Return on Knowledge (ROK: revenue/cost) and ROI (revenue-investment cost/investment cost). The outputs from a KVA analysis become the input into the ROI models and real options analysis. By tracking the historical volatility of price and cost per unit as well as ROI, it is possible to establish risk (as compared to uncertainty) distributions, which is important for accurately estimating the value of real options.

The KVA method has been applied to numerous military core processes across the services. The KVA research has more recently provided a means for simplifying real options analysis for DoD processes. Current KVA research will provide a library of market comparable price and cost per unit of output estimates. This research will enable a more stable basis for comparisons of performance across core processes. This data also provides a means to establish risk distribution profiles for Integrated Risk Management approaches such as real options, and KVA currently is being linked directly to the Real Options Super Lattice Solver and Risk Simulator software for rapid adjustments to real options valuation projections.

Strategic Real Options Analysis

Suppose you are driving from point A to point B, and you only have or know one way to get there, a straight route. Further suppose that there is a lot of uncertainty as to what traffic conditions are like further down the road, and you risk



being stuck in traffic, and there's a 50% chance that will occur. Simulation will provide you the 50% figure. But so what? Knowing that half the time you will get stuck in traffic is valuable information, but the question now is, so what? Especially if you have to get to point B no matter what. However, if you had several alternate routes to get to point B, you can still drive the straight route but if you hit traffic, you can make a left, right, or U-turn, to get around congestion, mitigating the risk, and getting you to point B faster and safer; that is, you have options. So, how much is such a strategic road map or global positioning satellite map worth to you? In military situations with high risk, real options can help you create strategies to mitigate these risks. In fact, businesses and the military have been doing real options for hundreds of years without realizing it. For instance, in the military, we call it courses of action or analysis of alternatives—do we take Hill A so that it provides us the option and ability to take Hill B and Valley C, or how should we take Valley C or do we avoid taking Valley C altogether, and so forth. A piece that is missing is the more formal structure and subsequent analytics that real options analysis provides. Using real options analysis, we can quantify and value each strategic pathway and frame strategies that will hedge or mitigate (and sometimes take advantage of) risk.

In the past, corporate investment decisions were cut-and-dried. Buy a new machine that is more efficient, makes more products costing a certain amount, and if the benefits outweigh the costs, execute the investment. Hire a larger pool of sales associates, expand the current geographical area, and if the marginal increase in forecast sales revenues exceeds the additional salary and implementation costs, start hiring. Need a new manufacturing plant? Show that the construction costs can be recouped quickly and easily by the increase in revenues it will generate through new and more improved products, and the initiative is approved. However, real-life conditions are a lot more complicated. Your firm decides to go with a more automated 3D PDF software and Logistics Team Center environment, but multiple strategic paths exist. Which path do you choose? What are the options that you have? If you choose the wrong path, how do you get back on the right track? How do you value and prioritize the paths that exist? You are a venture capitalist firm with multiple business plans to consider. How do you value a start-up firm with no proven



track record? How do you structure a mutually beneficial investment deal? What is the optimal timing for a second or third round of financing?

Real options are useful not only in valuing a firm, asset, or investment decision through its strategic business options, but also as a strategic business tool in capital investment acquisition decisions. For instance, should the military invest millions in a new open architecture initiative, and if so, what are the values of the various strategies such an investment would enable, and how do we proceed? How does the military choose among several seemingly cashless, costly, and unprofitable information-technology infrastructure projects? Should it indulge its billions in a risky research and development initiative? The consequences of a wrong decision can be disastrous and lives could be at stake. In a traditional analysis, these questions cannot be answered with any certainty. In fact, some of the answers generated through the use of the traditional analysis are flawed because the model assumes a static, one-time decision-making process while the real options approach takes into consideration the strategic options certain projects create under uncertainty and a decision maker's flexibility in exercising or abandoning these options at different points in time, when the level of uncertainty has decreased or has become known over time.

Real options analysis can be used to frame strategies to mitigate risk, to value and find the optimal strategic pathway to pursue, and to generate options to enhance the value of the project while managing risks. Sample options include the option to expand, contract, or abandon, or sequential compound options (phased stage-gate options, options to wait and defer investments, proof of concept stages, milestone development, and research and development initiatives). Some sample applications in the military include applications of real options to acquisitions, Spiral Development, and various organizational configurations, as well as the importance of how Integrated and Open Architectures become real options multipliers. Under OMB Circular A-76, comparisons using real options analysis could be applied to enhance outsourcing comparisons between the government's Most Efficient Organization (MEO) and private sector alternatives. Real options can be used throughout JCIDS requirements generation and the Defense Acquisition System, for



example, DOTMLPF versus New Program/Service solution, Joint Integration, Analysis of Material Alternatives (AMA), Analysis of Alternatives (AoA), and Spiral Development. Many other applications exist in military decision analysis and portfolios.

Real Options: A Quick Peek Behind the Scenes

Real options analysis will be performed to determine the prospective value of the basic options over a multiyear period using KVA data as a platform. The strategic real options analysis is solved employing various methodologies, including the use of binomial lattices with a market-replicating portfolios approach, and backed up using a modified closed-form sequential compound option model. The value of a compound option is based on the value of another option. That is, the underlying variable for the compound option is another option, and the compound option can be either sequential in nature or simultaneous. Solving such a model requires programming capabilities. This subsection is meant as a quick peek into the math underlying a very basic closed-form compound option. This section is only a preview of the detailed modeling techniques used in the current analysis and should not be assumed to be the final word.

For instance, we first start by solving for the critical value of I, an iterative component in the model using:

$$X_2 = Ie^{-q(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1) + (r - q + \sigma^2/2)(T_2 - t_1)}{\sigma\sqrt{(T_2 - t_1)}}\right) - X_1e^{-r(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1) + (r - q - \sigma^2/2)(T_2 - t_1)}{\sigma\sqrt{(T_2 - t_1)}}\right)$$



Then, solve recursively for the value I above and input it into the model:

$$\begin{aligned}
 \text{Compound Option} &= Se^{-qt_2} \Omega \left[\frac{\ln(S / X_1) + (r - q + \sigma^2 / 2)T_2}{\sigma\sqrt{T_2}}; \right. \\
 &\quad \left. \frac{\ln(S / I) + (r - q + \sigma^2 / 2)t_1}{\sigma\sqrt{t_1}}; \sqrt{t_1 / T_2} \right] \\
 &- X_1 e^{-rt_2} \Omega \left[\frac{\ln(S / X_1) + (r - q + \sigma^2 / 2)T_2 - \sigma\sqrt{T_2}}{\sigma\sqrt{T_2}}; \right. \\
 &\quad \left. \frac{\ln(S / I) + (r - q + \sigma^2 / 2)t_1 - \sigma\sqrt{t_1}; \sqrt{t_1 / T_2}}{\sigma\sqrt{t_1}} \right] \\
 &- X_2 e^{-rt_1} \Phi \left[\frac{\ln(S / I) + (r - q + \sigma^2 / 2)t_1 - \sigma\sqrt{t_1}}{\sigma\sqrt{t_1}} \right]
 \end{aligned}$$

The model is then applied to a sequential problem where future phase options depend on previous phase options (e.g., Phase II depends on Phase I's successful implementation).

The following are definitions of variables:

- S present value of future cash flows (\$)
- r risk-free rate (%)
- σ volatility (%)
- Φ cumulative standard-normal
- q continuous dividend payout (%)
- I critical value solved recursively
- Φ cumulative bivariate-normal
- X_1 strike for the underlying (\$)
- X_2 strike for the option on the option (\$)
- t_1 expiration date for the option on the option
- T_2 expiration date for the underlying option

The preceding closed-form differential equation models are then verified using the risk-neutral market-replicating portfolio approach assuming a sequential



compound option. In solving the market-replicating approach, we use the following functional forms (Mun, 2016):

- Hedge ratio (h):

$$h_{i-1} = \frac{C_{up} - C_{down}}{S_{up} - S_{down}}$$

- Debt load (D):

$$D_{i-1} = S_i(h_{i-1}) - C_i$$

- Call value (C) at node i :

$$C_i = S_i(h_i) - D_i e^{-rf(\delta)}$$

- Risk-adjusted probability (q):

$$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}} \text{ obtained assuming}$$

$$S_{i-1} = q_i S_{up} + (1 - q_i) S_{down}$$

- This means that

$$S_{i-1} = q_i S_{up} + S_{down} - q_i S_{down}$$

and

$$q_i [S_{up} - S_{down}] = S_{i-1} - S_{down},$$

so we get

$$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}}$$

Portfolio Optimization

In most decisions, there are variables over which leadership has control, such as how much to establish supply lines, modernize a ship, use network centrality to gather intelligence, and so on. Similarly, business leaders have options in what they charge for a product or how much to invest in a project or which projects they should



choose in a portfolio when they are constrained by budgets or resources. These decisions could also include allocating financial resources, building or expanding facilities, managing inventories, and determining product-mix strategies. Such decisions might involve thousands or millions of potential alternatives. Considering and evaluating each of them would be impractical or even impossible. These controlled variables are called decision variables. Finding the optimal values for decision variables can make the difference between reaching an important goal and missing that goal. An optimization model can provide valuable assistance in incorporating relevant variables when analyzing decisions, and finding the best solutions for making decisions. Optimization models often provide insights that intuition alone cannot. An optimization model has three major elements: decision variables, constraints, and an objective. In short, the optimization methodology finds the best combination or permutation of decision variables (e.g., best way to deploy troops, build ships, which projects to execute) in every conceivable way such that the objective is maximized (e.g., strategic value, enemy assets destroyed, return on investment) or minimized (e.g., risk and costs) while still satisfying the constraints (e.g., time, budget, and resources).

Obtaining optimal values generally requires that you search in an iterative or ad hoc fashion. This search involves running one iteration for an initial set of values, analyzing the results, changing one or more values, rerunning the model, and repeating the process until you find a satisfactory solution. This process can be very tedious and time consuming even for small models, and it is often unclear how to adjust the values from one iteration to the next. A more rigorous method systematically enumerates all possible alternatives. This approach guarantees optimal solutions if the model is correctly specified. Suppose that an optimization model depends on only two decision variables. If each variable has 10 possible values, trying each combination requires 100 iterations (102 alternatives). If each iteration is very short (e.g., 2 seconds), then the entire process could be done in approximately three minutes of computer time. However, instead of two decision variables, consider six, then consider that trying all combinations requires 1,000,000 iterations (106 alternatives). It is easily possible for complete enumeration to take



many years to carry out. Therefore, optimization has always been a fantasy until now; with the advent of sophisticated software and computing power, coupled with smart heuristics and algorithms, such analyses can be done within minutes.

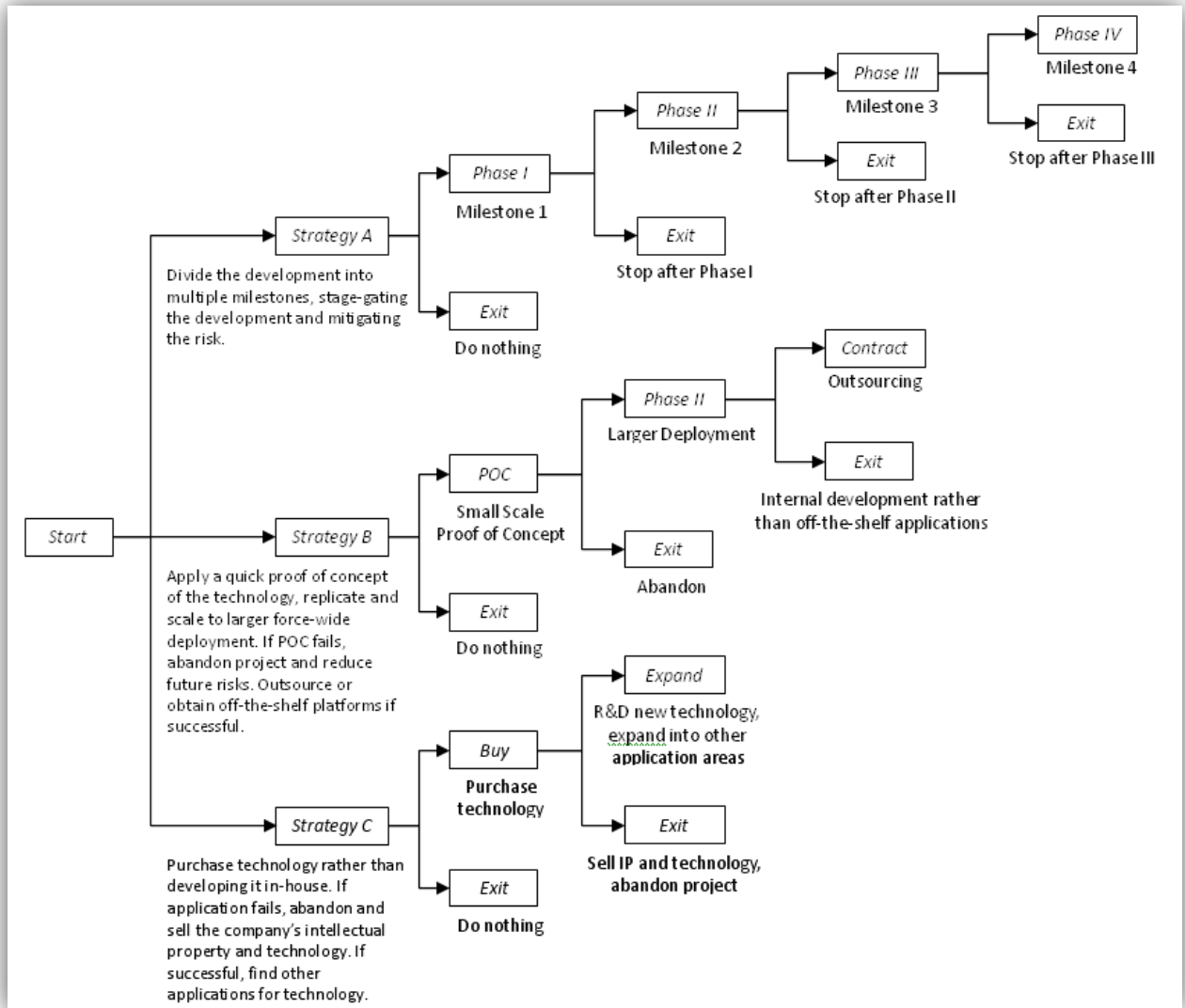


Figure A.5. Example Real Options Framing

Figures A.6, A.7, and A.8 illustrate a sample portfolio analysis where in the first case, there are 20 total projects to choose from (if all projects were executed, it would cost \$10.2 billion), and where each project has its own returns on investment or benefits measure, cost, strategic ranking, comprehensive, and tactical and total military scores (these were obtained from field commanders through the Delphi

method to elicit their thoughts about how strategic a particular project or initiative will be, and so forth). The constraints are full-time equivalence resources, budget, and strategic score. In other words, there are 20 projects or initiatives to choose from, and we want to select the top 10, subject to having enough money to pay for them and the people to do the work; yet we also want the most strategic portfolio possible. All the while, Monte Carlo simulation, real options, and forecasting methodologies are applied in the optimization model (e.g., each project's values shown in Figure A.6 are linked from its own large model with simulation and forecasting methodologies applied, and the best strategy for each project is chosen using real options analysis, or perhaps the projects shown are nested within one another; for instance, you cannot exercise Project 2 unless you execute Project 1, but you can only exercise Project 1 without having to do Project 2, and so forth). The results are shown in Figure A.6.

Figure A.7 shows the optimization process done in series, while relaxing some of the constraints. For instance, what would be the best portfolio and the strategic outcome if a budget of \$3.8 billion was imposed? What if it was increased to \$4.8 billion, \$5.8 billion, and so forth? The efficient frontiers depicted in Figure A.7 illustrate the best combination and permutation of projects in the optimal portfolio. Each point on the frontier is a portfolio of various combinations of projects that provides the best allocation possible given the requirements and constraints. Finally, Figure A.8 shows the top 10 projects that were chosen and how the total budget is best and most optimally allocated to provide the best and most well-balanced portfolio.



Project Name	ENPV	Benefits	Cost	Strategy Ranking	Return to Rank Ratio	Profitability Index	Selection	Comprehensive Score	Tactical Score	FTE Resources	Military Score
Project 1	\$458.00	\$150.76	\$1,732.44	1.20	381.67	1.09	0	8.10	2.31	1.20	1.98
Project 2	\$1,954.00	\$245.00	\$859.00	9.80	199.39	1.29	1	1.27	4.83	2.50	1.76
Project 3	\$1,599.00	\$458.00	\$1,845.00	9.70	164.85	1.25	0	9.88	4.75	3.60	2.77
Project 4	\$2,251.00	\$529.00	\$1,645.00	4.50	500.22	1.32	0	8.83	1.61	4.50	2.07
Project 5	\$849.00	\$564.00	\$458.00	10.90	77.89	2.23	0	5.02	6.25	5.50	2.94
Project 6	\$758.00	\$135.00	\$52.00	7.40	102.43	3.60	1	3.64	5.79	9.20	3.26
Project 7	\$2,845.00	\$311.00	\$758.00	19.80	143.69	1.41	1	5.27	6.47	12.50	4.04
Project 8	\$1,235.00	\$754.00	\$115.00	7.50	164.67	7.56	1	9.80	7.16	5.30	3.63
Project 9	\$1,945.00	\$198.00	\$125.00	10.80	180.09	2.58	1	5.68	2.39	6.30	2.16
Project 10	\$2,250.00	\$785.00	\$458.00	8.50	264.71	2.71	1	8.29	4.41	4.50	2.67
Project 11	\$549.00	\$35.00	\$45.00	4.80	114.38	1.78	0	7.52	4.65	4.90	2.75
Project 12	\$525.00	\$75.00	\$105.00	5.90	88.98	1.71	0	5.54	5.09	5.20	2.69
Project 13	\$516.00	\$451.00	\$48.00	2.80	184.29	10.40	0	2.51	2.17	4.60	1.66
Project 14	\$499.00	\$458.00	\$351.00	9.40	53.09	2.30	1	9.41	9.49	9.90	4.85
Project 15	\$859.00	\$125.00	\$421.00	6.50	132.15	1.30	1	6.91	9.62	7.20	4.25
Project 16	\$884.00	\$458.00	\$124.00	3.90	226.67	4.69	1	7.06	9.98	7.50	4.46
Project 17	\$956.00	\$124.00	\$521.00	15.40	62.08	1.24	1	1.25	2.50	8.60	2.07
Project 18	\$854.00	\$164.00	\$512.00	21.00	40.67	1.32	0	3.09	2.90	4.30	1.70
Project 19	\$195.00	\$45.00	\$5.00	1.20	162.50	10.00	0	5.25	1.22	4.10	1.86
Project 20	\$210.00	\$85.00	\$21.00	1.00	210.00	5.05	0	2.01	4.06	5.20	2.50
Total	\$14,185.00		\$3,784.00	99.00			10	58.58	62.64	73.50	33.15
Profit/Rank	\$143.28										
Profit*Score	\$470,235.60	Maximize	<= \$3800	<= 100			x <= 10				<= 80

Figure A.6. Portfolio Optimization and Allocation



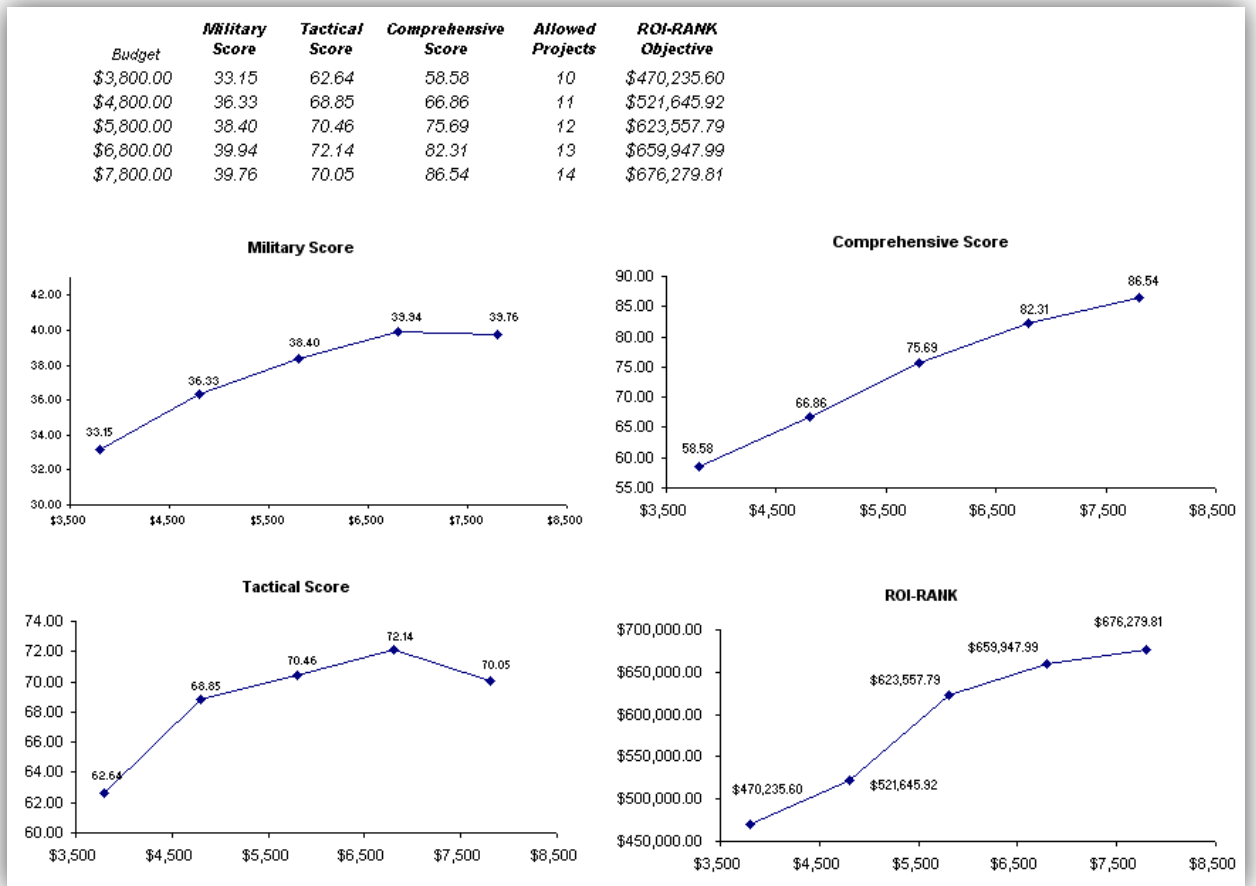


Figure A.7. Efficient Frontiers of Portfolios

ASSET ALLOCATION OPTIMIZATION MODEL										
Asset Class Description	Annualized Returns	Volatility Risk	Allocation Weights	Required Minimum Allocation	Required Maximum Allocation	Return to Risk Ratio	Returns Ranking (Hi-Lo)	Risk Ranking (Lo-Hi)	Return to Risk Ranking (Hi-Lo)	Allocation Ranking (Hi-Lo)
Selected Project 1	10.50%	12.38%	11.10%	5.00%	35.00%	0.8483	9	2	7	4
Selected Project 2	11.12%	16.36%	6.74%	5.00%	35.00%	0.6799	7	8	10	10
Selected Project 3	11.77%	15.81%	7.63%	5.00%	35.00%	0.7445	6	7	9	9
Selected Project 4	10.77%	12.33%	11.49%	5.00%	35.00%	0.8738	8	1	5	3
Selected Project 5	13.49%	13.35%	12.26%	5.00%	35.00%	1.0102	5	4	2	2
Selected Project 6	14.24%	14.53%	10.94%	5.00%	35.00%	0.9800	3	6	3	5
Selected Project 7	15.60%	14.30%	12.36%	5.00%	35.00%	1.0908	1	5	1	1
Selected Project 8	14.95%	16.64%	8.75%	5.00%	35.00%	0.8983	2	10	4	7
Selected Project 9	14.15%	16.56%	8.36%	5.00%	35.00%	0.8545	4	9	6	8
Selected Project 10	10.08%	12.55%	10.37%	5.00%	35.00%	0.8027	10	3	8	6
Portfolio Total	12.7270%	4.54%	100.00%							
Return to Risk Ratio	2.8021									

Figure A.8. Portfolio Optimization (Continuous Allocation of Funds)



Integrated Risk Management Framework

We are now able to put all the pieces together into an integrated risk management framework and see how these different techniques are related in a risk analysis and risk management context. This framework comprises eight distinct phases of a successful and comprehensive risk analysis implementation, going from a qualitative management screening process to creating clear and concise reports for management. The process was developed by the author (Mun) based on previous successful implementations of risk analysis, forecasting, real options, KVA cash-flow estimates, valuation, and optimization projects both in the consulting arena and in industry-specific problems. These phases can be performed either in isolation or together in sequence for a more robust integrated analysis.

Figure A.9 shows the integrated risk management process up close. We can segregate the process into the following eight simple steps:

1. Qualitative management screening
2. Time-series and regression forecasting
3. Base case KVA and net present value analysis
4. Monte Carlo simulation
5. Real options problem framing
6. Real options modeling and analysis
7. Portfolio and resource optimization
8. Reporting and update analysis

1. Qualitative Management Screening

Qualitative management screening is the first step in any integrated risk management process. Decision makers have to decide which projects, assets, initiatives, or strategies are viable for further analysis, in accordance with the organization's mission, vision, goal, or overall business strategy. The organization's mission, vision, goal, or overall business strategy may include strategies and tactics,



and competitive advantage, technical, acquisition, growth, synergistic, or global threat issues. That is, the initial list of projects should be qualified in terms of meeting the leadership's agenda. Often the most valuable insight is created as leaders frame the complete problem to be resolved. This is where the various risks to the organization are identified and fleshed out.

2. Time-Series and Regression Forecasting

The future is then forecasted using time-series analysis, stochastic forecasting, or multivariate regression analysis if historical or comparable data exist. Otherwise, other qualitative forecasting methods may be used (subjective guesses, growth rate assumptions, expert opinions, Delphi method, and so forth).

3. Base Case KVA and Net Present Value Analysis

For each project that passes the initial qualitative screens, a KVA-based discounted cash flow model is created. This model serves as the base case analysis where a net present value and ROI are calculated for each project, using the forecasted values in the previous step. This step also applies if only a single project is under evaluation. This net present value is calculated with the traditional approach of using the forecast revenues and costs and discounting the net of these revenues and costs at an appropriate risk-adjusted rate. The ROI and other financial metrics are generated here.

4. Monte Carlo Simulation

Because the static discounted cash flow produces only a single-point estimate result, there is oftentimes little confidence in its accuracy given that future events that affect forecast cash flows are highly uncertain. To better estimate the actual value of a particular project, Monte Carlo simulation should be employed next. Usually a sensitivity analysis is first performed on the discounted cash flow model; that is, setting the net present value or ROI as the resulting variable, we can change each of its precedent variables and note the change in the resulting variable. Precedent variables include revenues, costs, tax rates, discount rates, capital expenditures, depreciation, and so forth, which ultimately flow through the model to affect the net present value or ROI figure. By tracing back all these precedent



variables, we can change each one by a preset amount and see the effect on the resulting net present value. A graphical representation can then be created in Risk Simulator, which is often called a tornado chart because of its shape, where the most sensitive precedent variables are listed first, in descending order of magnitude. Armed with this information, the analyst can then decide which key variables are highly uncertain in the future and which are deterministic. The uncertain key variables that drive the net present value, and, hence, the decision, are called critical success drivers. These critical success drivers are prime candidates for Monte Carlo simulation. Because some of these critical success drivers may be correlated, a correlated and multidimensional Monte Carlo simulation may be required. Typically, these correlations can be obtained through historical data. Running correlated simulations provides a much closer approximation to the variables' real-life behaviors.

5. Real Options Problem Framing

The question now is, after quantifying risks in the previous step, what is next? The risk information obtained needs to be converted somehow into actionable intelligence. Risk has been quantified using Monte Carlo simulation, but so what? And what do we do about it? The answer is to use real options analysis to hedge these risks, to value these risks, and to get in a position to take advantage of the risks. The first step in real options is to generate a strategic map through the process of framing the problem. Based on the overall problem identification occurring during the initial qualitative management screening process, certain strategic optionalities would have become apparent for each particular project. The strategic optionalities may include, among other things, the option to expand, contract, abandon, switch, choose, and so forth. Based on the identification of strategic optionalities that exist for each project or at each stage of the project, the analyst can then choose an item from a list of options to analyze in more detail. Real options are added to the projects to hedge downside risks and to take advantage of upside swings.



6. Real Options Modeling and Analysis

Through the use of Monte Carlo simulation, the resulting stochastic discounted cash flow model will have a distribution of values. Thus, simulation models, analyzes, and quantifies the various risks and uncertainties of each project. The result is a distribution of the NPVs and the project's volatility. In real options, we assume that the underlying variable is the future profitability of the project, which is the future cash flow series. An implied volatility of the future free cash flow or underlying variable can be calculated through the results of a Monte Carlo simulation, as previously performed. Usually, the volatility is measured as the standard deviation of the logarithmic returns on the free cash flow stream. In addition, the present value of future cash flows for the base case discounted cash flow model is used as the initial underlying asset value in real options modeling. Using these inputs, real options analysis is performed to obtain the projects' strategic option values.

7. Portfolio and Resource Optimization

Portfolio optimization is an optional step in the analysis. If the analysis is done on multiple projects, decision makers should view the results as a portfolio of rolled-up projects because the projects are in most cases correlated with one another, and viewing them individually will not present the true picture. As organizations do not only have single projects, portfolio optimization is crucial. Given that certain projects are related to others, there are opportunities for hedging and diversifying risks through a portfolio. Because firms have limited budgets and time and resource constraints, and also have requirements for overall levels of returns, risk tolerances, and so forth; portfolio optimization takes into account these factors to create an optimal portfolio mix. The analysis will provide the optimal allocation of investments across multiple projects.

8. Reporting and Update Analysis

The analysis is not complete until reports can be generated. Both the results and the process should be presented. Clear, concise, and precise explanations transform a difficult black-box set of analytics into transparent steps. Decision



makers will never accept results coming from black boxes if they do not understand where the assumptions or data originate and what types of mathematical or analytical massaging takes place. Risk analysis assumes that the future is uncertain and that decision makers have the right to make midcourse corrections when these uncertainties become resolved or risks become known; the analysis is usually done ahead of time and thus ahead of such uncertainty and risks. Therefore, when these risks become known over the passage of time, actions, and events, the analysis should be revisited to incorporate the decisions made or revising any input assumptions. Sometimes, for long-horizon projects, several iterations of the real options analysis should be performed, where future iterations are updated with the latest data and assumptions. Understanding the steps required to undertake an integrated risk management analysis is important because it provides insight, not only into the methodology itself, but also into how it evolves from traditional analyses, showing where the traditional approach ends and where the new analytics start.

Conclusion

Hopefully it has now become evident that the DoD leadership can take advantage of more advanced analytical procedures when making strategic investment decisions and when managing portfolios of projects. In the past, due to the lack of technological maturity, this was extremely difficult, and, hence, businesses and the government had to resort to experience and managing by gut feel. Nowadays, however, with the assistance of technology and more mature methodologies, there is every reason to take the analysis a step further. Corporations such as 3M, Airbus, AT&T, Boeing, BP, Chevron, Johnson & Johnson, Motorola, and many others have already been successfully using these techniques for years, and the military can and should follow suit. The relevant software applications, books, case studies, and public seminars have been created, and case studies have already been developed for the U.S. Navy. The only barrier to implementation, simply put, is the lack of exposure to the potential benefits of the methods. Many in the military have not seen or even heard of these new concepts.



This primer, if it is successful, serves to reveal the potential benefits of these analytical techniques and tools that can complement what leadership is already doing. In order to be ready for the challenges of the 21st century, and to create a highly effective and flexible military force, strategic real options, KVA, and risk analysis are available to aid leadership with critical decision making. Real options and KVA are tools that will help ensure maximum strategic flexibility and analysis of alternatives where risks must be considered.



Integrated Risk Management Process

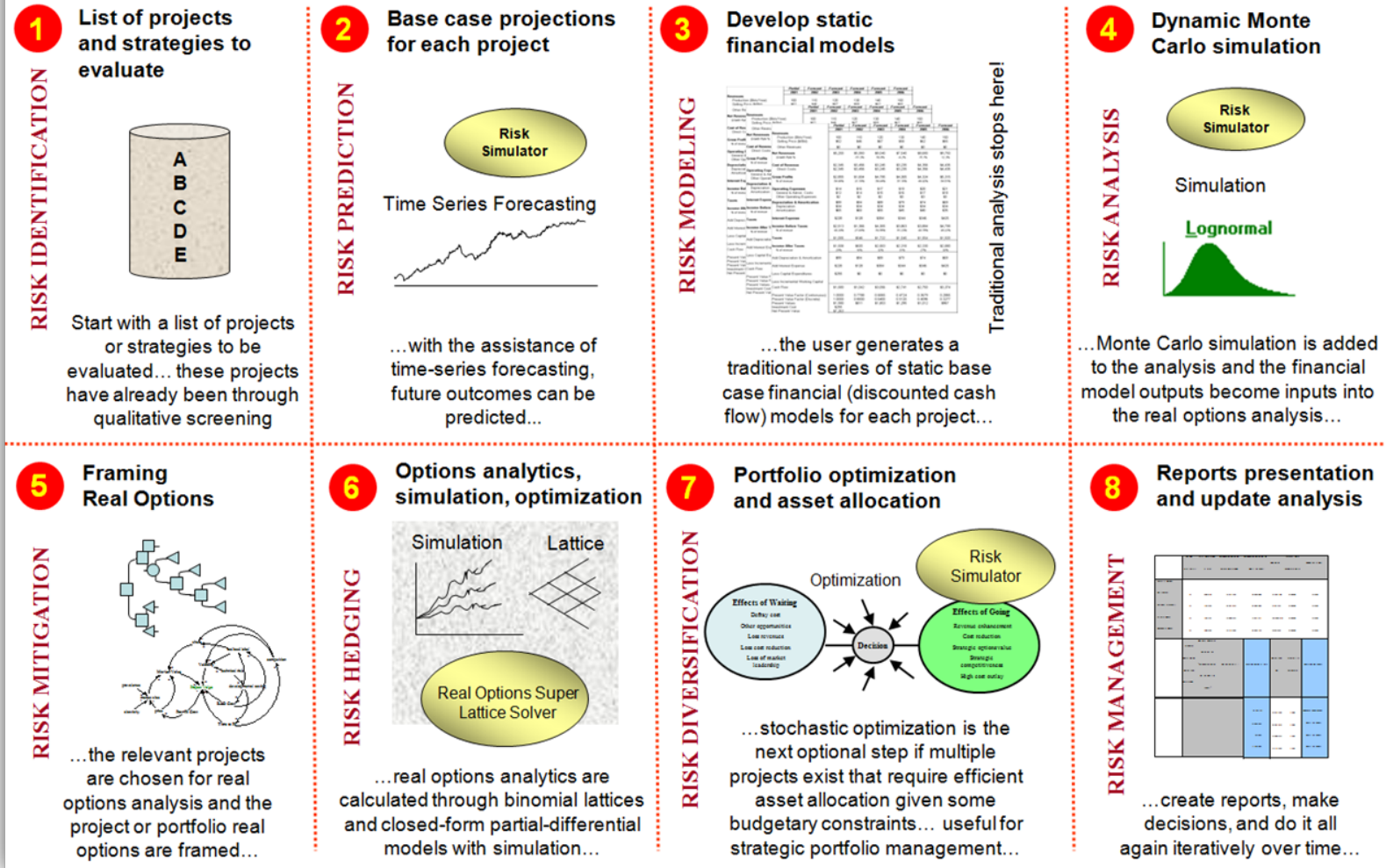


Figure A.9. Integrated Risk Management Process

Appendix 2—Understanding Probability Distributions

This appendix demonstrates the power of Monte Carlo risk simulation, but in order to get started with simulation, one first needs to understand the concept of probability distributions. This appendix continues with the use of the author's Risk Simulator software and shows how simulation can be very easily and effortlessly implemented in an existing Excel model.

To begin to understand probability, consider the following example. You want to look at the distribution of nonexempt wages within one department of a large company. First, you gather raw data—in this case, the wages of each nonexempt employee in the department. Second, you organize the data into a meaningful format and plot the data as a frequency distribution on a chart. To create a frequency distribution, you divide the wages into group intervals and list these intervals on the chart's horizontal axis. Then you list the number or frequency of employees in each interval on the chart's vertical axis. Now you can easily see the distribution of nonexempt wages within the department.

A glance at Figure A.10 reveals that the employees earn from \$7.00 to \$9.00 per hour. You can chart this data as a probability distribution. A probability distribution shows the number of employees in each interval as a fraction of the total number of employees. To create a probability distribution, you divide the number of employees in each interval by the total number of employees and list the results on the chart's vertical axis.





Figure A.10. Frequency Histogram I

Figure A.11 shows the number of employees in each wage group as a fraction of all employees; you can estimate the likelihood or probability that an employee drawn at random from the whole group earns a wage within a given interval. For example, assuming the same conditions exist at the time the sample was taken, the probability is 0.20 (a one in five chance) that an employee drawn at random from the whole group earns \$8.50 an hour.

Probability distributions are either discrete or continuous. *Discrete probability distributions* describe distinct values, usually integers, with no intermediate values and are shown as a series of vertical bars. A discrete distribution, for example, might describe the number of heads in four flips of a coin as 0, 1, 2, 3, or 4. *Continuous probability distributions* are actually mathematical abstractions because they assume the existence of every possible intermediate value between two numbers; that is, a continuous distribution assumes there is an infinite number of values between any two points in the distribution. However, in many situations, you can effectively use a continuous distribution to approximate a discrete distribution even though the continuous model does not necessarily describe the situation exactly.

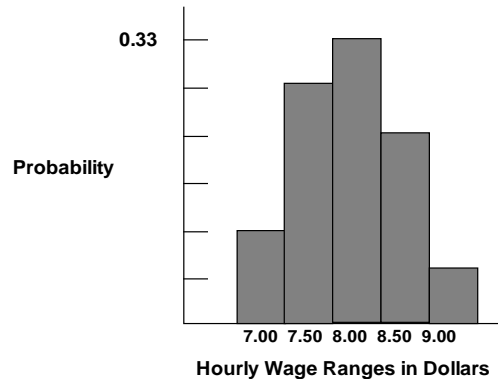


Figure A.11. Frequency Histogram II

Selecting a Probability Distribution

Plotting data is one method for selecting a probability distribution. The following steps provide another process for selecting probability distributions that best describe the uncertain variables in your spreadsheets.

To select the correct probability distribution, use the following steps:

- Look at the variable in question. List everything you know about the conditions surrounding this variable. You might be able to gather valuable information about the uncertain variable from historical data. If historical data are not available, use your own judgment, based on experience, listing everything you know about the uncertain variable.
- Review the descriptions of the probability distributions.
- Select the distribution that characterizes this variable. A distribution characterizes a variable when the conditions of the distribution match those of the variable.

Alternatively, if you have historical, comparable, contemporaneous, or forecast data, you can use Risk Simulator's distributional fitting modules to find the best statistical fit for your existing data. This fitting process will apply some advanced statistical techniques to find the best distribution and its relevant parameters that describe the data.

Probability Density Functions, Cumulative Distribution Functions, and Probability Mass Functions

In mathematics and Monte Carlo simulation, a probability density function (PDF) represents a *continuous* probability distribution in terms of integrals. If a probability distribution has a density of $f(x)$, then intuitively the infinitesimal interval of $[x, x + dx]$ has a probability of $f(x) dx$. The PDF therefore can be seen as a smoothed version of a probability histogram; that is, by providing an empirically large sample of a continuous random variable repeatedly, the histogram using very narrow ranges will resemble the random variable's PDF. The probability of the interval between $[a, b]$ is given by $\int_a^b f(x)dx$, which means that the total integral of the function f must be 1.0. It is a common mistake to think of $f(a)$ as the probability of a , when, in fact, $f(a)$ can sometimes be larger than 1—consider a uniform distribution between 0.0 and 0.5. The random variable x within this distribution will have $f(x)$ greater than 1. The probability, in reality, is the function $f(x)dx$ discussed previously, where dx is an infinitesimal amount.

The cumulative distribution function (CDF) is denoted as $F(x) = P(X \leq x)$ indicating the probability of X taking on a less than or equal value to x . Every CDF is monotonically increasing, is continuous from the right, and at the limits, has the following properties: $\lim_{x \rightarrow -\infty} F(x) = 0$ and $\lim_{x \rightarrow +\infty} F(x) = 1$. Further, the CDF is related to the PDF by

$$F(b) - F(a) = P(a \leq X \leq b) = \int_a^b f(x)dx, \text{ where the PDF function } f \text{ is the derivative of the CDF}$$

function F .

In probability theory, a probability mass function, or PMF, gives the probability that a *discrete* random variable is exactly equal to some value. The PMF differs from the PDF in that the values of the latter, defined only for continuous random variables, are not probabilities; rather, its integral over a set of possible values of the random variable is a probability. A random variable is discrete if its probability distribution is discrete and can be characterized by a PMF. Therefore, X is a discrete



random variable if $\sum_u P(X = u) = 1$ as u runs through all possible values of the random variable X .

Normal Distribution

The normal distribution is the most important distribution in probability theory because it describes many natural phenomena, such as people's IQs or heights. Decision makers can use the normal distribution to describe uncertain variables such as the inflation rate or the future price of gasoline.

The three conditions underlying the normal distribution are

- Some value of the uncertain variable is the most likely (the mean of the distribution).
- The uncertain variable could as likely be above the mean as it could be below the mean (symmetrical about the mean).
- The uncertain variable is more likely in the vicinity of the mean than further away.



The mathematical constructs for the normal distribution are as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \text{ for all values of } x \text{ and } \mu; \text{ while } \sigma > 0$$

Mean = μ

Standard Deviation = σ

Skewness = 0 (this applies to all inputs of mean and standard deviation)

Excess Kurtosis = 0 (this applies to all inputs of mean and standard deviation)

Mean (μ) and standard deviation (σ) are the distributional parameters.

Input requirements: Standard deviation > 0 and can be any positive value whereas mean can be any value

PERT Distribution

The PERT distribution is widely used in project and program management to define the worst-case, nominal-case, and best-case scenarios of project completion time. It is related to the beta and triangular distributions. PERT distribution can be used to identify risks in project and cost models based on the likelihood of meeting targets and goals across any number of project components using minimum, most likely, and maximum values, but it is designed to generate a distribution that more closely resembles realistic probability distributions. The PERT distribution can provide a close fit to the normal or lognormal distributions. Like the triangular distribution, the PERT distribution emphasizes the most likely value over the minimum and maximum estimates. However, unlike the triangular distribution, the PERT distribution constructs a smooth curve that places progressively more emphasis on values around (near) the most likely value, in favor of values around the edges. In practice, this means that we trust the estimate for the most likely value, and we believe that even if it is not exactly accurate (as estimates seldom are), we have an expectation that the resulting value will be close to that estimate. Assuming that many real-world phenomena are normally distributed, the appeal of the PERT distribution is that it produces a curve similar to the normal curve in shape, without



knowing the precise parameters of the related normal curve. Minimum, most likely, and maximum are the distributional parameters.

The mathematical constructs for the PERT distribution are shown below:

$$f(x) = \frac{(x - \text{Min})^{A1-1} (\text{Max} - x)^{A2-1}}{B(A1, A2)(\text{Max} - \text{Min})^{A1+A2-1}}$$

$$\text{where } A1 = 6 \left[\frac{\text{Min} + 4(\text{Likely}) + \text{Max}}{6} - \text{Min} \right] \text{ and } A2 = 6 \left[\text{Max} - \frac{\text{Min} + 4(\text{Likely}) + \text{Max}}{6} \right]$$

and B is the Beta function

$$\text{Mean} = \frac{\text{Min} + 4\text{Mode} + \text{Max}}{6}$$

$$\text{Standard Deviation} = \sqrt{\frac{(\mu - \text{Min})(\text{Max} - \mu)}{7}}$$

$$\text{Skew} = \sqrt{\frac{7}{(\mu - \text{Min})(\text{Max} - \mu)}} \left(\frac{\text{Min} + \text{Max} - 2\mu}{4} \right)$$

Excess Kurtosis is a complex function and cannot be readily computed.

Input requirements: $\text{Min} \leq \text{Most Likely} \leq \text{Max}$ and can be positive, negative, or zero.

Triangular Distribution

The triangular distribution describes a situation where you know the minimum, maximum, and most likely values to occur. For example, you could describe the number of cars sold per week when past sales show the minimum, maximum, and usual number of cars sold.

The three conditions underlying the triangular distribution are

- The minimum number of items is fixed.
- The maximum number of items is fixed.
- The most likely number of items falls between the minimum and maximum values, forming a triangular-shaped distribution, which shows that values



near the minimum and maximum are less likely to occur than those near the most-likely value.

The mathematical constructs for the triangular distribution are as follows:

$$f(x) = \begin{cases} \frac{2(x - \text{Min})}{(\text{Max} - \text{Min})(\text{Likely} - \text{Min})} & \text{for } \text{Min} < x < \text{Likely} \\ \frac{2(\text{Max} - x)}{(\text{Max} - \text{Min})(\text{Max} - \text{Likely})} & \text{for } \text{Likely} < x < \text{Max} \end{cases}$$

$$\text{Mean} = \frac{1}{3}(\text{Min} + \text{Likely} + \text{Max})$$

$$\text{Standard Deviation} = \sqrt{\frac{1}{18}(\text{Min}^2 + \text{Likely}^2 + \text{Max}^2 - \text{MinMax} - \text{MinLikely} - \text{MaxLikely})}$$

$$\text{Skewness} = \frac{\sqrt{2}(\text{Min} + \text{Max} - 2\text{Likely})(2\text{Min} - \text{Max} - \text{Likely})(\text{Min} - 2\text{Max} + \text{Likely})}{5(\text{Min}^2 + \text{Max}^2 + \text{Likely}^2 - \text{MinMax} - \text{MinLikely} - \text{MaxLikely})^{3/2}}$$

Excess Kurtosis = -0.6 (this applies to all inputs of Min, Max, and Likely)

Minimum (Min), most likely (Likely) and maximum (Max) are the parameters.

Input requirements:

Min ≤ Most Likely ≤ Max and can take any value.

However, Min < Max and can take any value.

Uniform Distribution

With the uniform distribution, all values fall between the minimum and maximum and occur with equal likelihood.

The three conditions underlying the uniform distribution are:

- The minimum value is fixed.
- The maximum value is fixed.
- All values between the minimum and maximum occur with equal likelihood.

The mathematical constructs for the uniform distribution are as follows:



$$f(x) = \frac{1}{Max - Min} \text{ for all values such that } Min < Max$$

$$\text{Mean} = \frac{Min + Max}{2}$$

$$\text{Standard Deviation} = \sqrt{\frac{(Max - Min)^2}{12}}$$

Skewness = 0 (this applies to all inputs of Min and Max)

Excess Kurtosis = -1.2 (this applies to all inputs of Min and Max)

Maximum value (Max) and minimum value (Min) are the distributional parameters.

Input requirements: Min < Max and can take any value.



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Appendix 3—Distributional Fitting Algorithms

Generally speaking, distributional fitting answers the questions: Which distribution does an analyst or engineer use for a particular input variable in a model? What are the relevant distributional parameters? Following are additional methods of distributional fitting available in Risk Simulator:

- Akaike Information Criterion (AIC). Rewards goodness-of-fit but also includes a penalty that is an increasing function of the number of estimated parameters (although AIC penalizes the number of parameters less strongly than other methods).
- Anderson–Darling (AD). When applied to testing if a normal distribution adequately describes a set of data, it is one of the most powerful statistical tools for detecting departures from normality and is powerful for testing normal tails. However, in non-normal distributions, this test lacks power compared to others.
- Kolmogorov–Smirnov (KS). A nonparametric test for the equality of continuous probability distributions that can be used to compare a sample with a reference probability distribution, making it useful for testing abnormally shaped distributions and non-normal distributions.
- Kuiper’s Statistic (K). Related to the KS test making it as sensitive in the tails as at the median and also making it invariant under cyclic transformations of the independent variable, rendering it invaluable when testing for cyclic variations over time. In comparison, the AD test provides equal sensitivity at the tails as the median, but it does not provide the cyclic invariance.
- Schwarz/Bayes Information Criterion (SC/BIC). The SC/BIC test introduces a penalty term for the number of parameters in the model with a larger penalty than AIC.



The null hypothesis being tested is such that the fitted distribution is the same distribution as the population from which the sample data to be fitted comes. Thus, if the computed p -value is lower than a critical alpha level (typically 0.10 or 0.05), then the distribution is the wrong distribution (reject the null hypothesis). Conversely, the higher the p -value, the better the distribution fits the data (do not reject the null hypothesis, which means the fitted distribution is the correct distribution, or null hypothesis of $H_0: \text{Error} = 0$, where error is defined as the difference between the empirical data and the theoretical distribution). Roughly, you can think of p -value as a percentage explained; that is, for example, if the computed p -value of a fitted normal distribution is 0.9996, then setting a normal distribution with the fitted mean and standard deviation explains about 99.96% of the variation in the data, indicating an especially good fit. Both the results and the report show the test statistic, p -value, theoretical statistics (based on the selected distribution), empirical statistics (based on the raw data), the original data (to maintain a record of the data used), and the assumptions, complete with the relevant distributional parameters (i.e., if you selected the option to automatically generate assumptions and if a simulation profile already exists). The results also rank all the selected distributions and how well they fit the data.

Goodness-of-Fit Tests

Several statistical tests exist for deciding if a sample set of data comes from a specific distribution. The most commonly used are the Kolmogorov–Smirnov test and the Chi-Square test. Each test has its advantages and disadvantages. The following sections detail the specifics of these tests as applied in distributional fitting in Monte Carlo simulation analysis. Other less powerful tests such as the Jacque-Bera and Wilkes-Shapiro are not used in Risk Simulator as these are parametric tests and their accuracy depends on the dataset being normal or near-normal. Therefore, the results of these tests are oftentimes suspect or yield inconsistent results.



Kolmogorov–Smirnov Test

The Kolmogorov–Smirnov (KS) test is based on the empirical distribution function of a sample dataset and belongs to a class of nonparametric tests. This nonparametric characteristic is the key to understanding the KS test, which simply means that the distribution of the KS test statistic does not depend on the underlying cumulative distribution function being tested. Nonparametric simply means no predefined distributional parameters are required. In other words, the KS test is applicable across a multitude of underlying distributions. Another advantage is that it is an exact test as compared to the Chi-Square test, which depends on an adequate sample size for the approximations to be valid. Despite these advantages, the KS test has several important limitations. It only applies to continuous distributions, and it tends to be more sensitive near the center of the distribution than at the distribution’s tails. Also, the distribution must be fully specified.

The hypothesis regarding the distributional form is rejected if the test statistic, KS, is greater than the critical value obtained from the table below. Notice that 0.03 to 0.05 are the most common levels of critical values (at the 1 percent, 5 percent, and 10 percent significance levels). Thus, any calculated KS statistic less than these critical values implies that the null hypothesis is not rejected and that the distribution is a good fit. There are several variations of these tables that use somewhat different scaling for the KS test statistic and critical regions. These alternative formulations should be equivalent, but it is necessary to ensure that the test statistic is calculated in a way that is consistent with how the critical values were tabulated. However, the rule of thumb is that a KS test statistic less than 0.03 or 0.05 indicates a good fit.

TWO-TAILED ALPHA LEVEL	KS CRITICAL
10%	0.03858
5%	0.04301
1%	0.05155

Chi-Square Test

The Chi-Square (CS) goodness-of-fit test is applied to binned data (i.e., data put into classes) and an attractive feature of the CS test is that it can be applied to



any univariate distribution for which you can calculate the cumulative distribution function. However, the values of the CS test statistic are dependent on how the data is binned, and the test requires a sufficient sample size in order for the CS approximation to be valid. This test is sensitive to the choice of bins. The test can be applied to discrete distributions such as the binomial and the Poisson, while the KS test is restricted to continuous distributions.

The null hypothesis is such that the dataset follows a specified distribution while the alternate hypothesis is that the dataset does not follow the specified distribution.

The test statistic follows a CS distribution with $(k - c)$ degrees of freedom where k is the number of nonempty cells and c is the number of estimated parameters (including location and scale parameters and shape parameters) for the distribution + 1. For example, for a three-parameter Weibull distribution, $c = 4$.

Again, as the null hypothesis is such that the data follow some specified distribution, when applied to distributional fitting in Risk Simulator, a low p -value (e.g., less than 0.10, 0.05, or 0.01) indicates a bad fit (the null hypothesis is rejected) while a high p -value indicates a statistically good fit.



Chi-Squared Goodness-of-Fit Test Sample Critical Values

Degrees of Freedom 23

ALPHA LEVEL	CUTOFF
10%	32.00690
5%	35.17246
1%	41.63840

Akaike Information Criterion, Anderson-Darling, Kuiper's Statistic, and Schwarz/Bayes Criterion

Following are additional methods of distributional fitting available in Risk Simulator:

- Akaike Information Criterion (AIC). Rewards goodness-of-fit but also includes a penalty that is an increasing function of the number of estimated parameters (although AIC penalizes the number of parameters less strongly than other methods).
- Anderson–Darling (AD). When applied to testing if a normal distribution adequately describes a set of data, it is one of the most powerful statistical tools for detecting departures from normality and is powerful for testing normal tails. However, in non-normal distributions, this test lacks power compared to others.
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The null hypothesis being tested is such that the fitted distribution is the same distribution as the population from which the sample data to be fitted comes. Thus, if the computed p -value is lower than a critical alpha level (typically .10 or .05), then the distribution is the wrong distribution (reject the null hypothesis). Conversely, the higher the p -value, the better the distribution fits the data (do not reject the null hypothesis, which means the fitted distribution is the correct distribution, or null hypothesis of H_0 : Error = 0, where error is defined as the difference between the empirical data and the theoretical distribution). Roughly, you can think of p -value as a percentage explained; that is, for example, if the computed p -value of a fitted normal distribution is 0.9727, then setting a normal distribution with the fitted mean and standard deviation explains about 97.27% of the variation in the data, indicating an especially good fit. Both the results and the report show the test statistic, p -value, theoretical statistics (based on the selected distribution), empirical statistics (based on the raw data), the original data (to maintain a record of the data used), and the assumptions complete with the relevant distributional parameters (i.e., if you selected the option to automatically generate assumptions and if a simulation profile already exists). The results also rank all the selected distributions and how well they fit the data.



Appendix 4—The Pitfalls of Forecasting: Outliers, Nonlinearity, Multicollinearity, Heteroskedasticity, Autocorrelation, and Structural Breaks

Forecasting is a balance between art and science. Using Risk Simulator can take care of the science, but it is almost impossible to take the art out of forecasting. In forecasting, experience and subject-matter expertise counts. One effective way to support this point is to look at some of the more common problems and violations of the required underlying assumptions of the data and forecast interpretation. Clearly there are many other technical issues, but the following list is sufficient to illustrate the pitfalls of forecasting and why sometimes the art (i.e., experience and expertise) is important:

- Out-of-Range Forecasts
- Nonlinearities
- Interactions
- Self-Selection Bias
- Survivorship Bias
- Control Variables
- Omitted Variables
- Redundant Variables
- Multicollinearity
- Bad-Fitting Model or Bad Goodness-of-Fit
- Error Measurements
- Structural Breaks
- Structural Shifts
- Model Errors (Granger Causality and Causality Loops)
- Autocorrelation
- Serial Correlation
- Leads and Lags
- Seasonality



- Cyclicalities
- Specification Errors and Incorrect Econometric Methods
- Micronumerosity
- Bad Data and Data Collection Errors
- Nonstationary Data, Random Walks, Non-Predictability, and Stochastic Processes (Brownian Motion, Mean-Reversion, Jump-Diffusion, Mixed Processes)
- Nonspherical and Dependent Errors
- Heteroskedasticity and Homoskedasticity
- Many other technical issues!

These errors predominantly apply to time-series data, cross-sectional data, and mixed-panel data. However, the following potential errors apply only to time-series data: Autocorrelation, Heteroskedasticity, and Nonstationarity.

Analysts sometimes use historical data to make *out-of-range forecasts* that, depending on the forecast variable, could be disastrous. Take, for example, the simple yet extreme case of a cricket. Did you know that if you caught some crickets, put them in a controlled lab environment, raised the ambient temperature, and counted the average number of chirps per minute, these chirps are relatively predictable? You might get a pretty good fit and a high R-squared value. So, the next time you go out on a date with your spouse or significant other and hear some crickets chirping on the side of the road, stop and count the number of chirps per minute. Then, using your regression forecast equation, you can approximate the temperature, and the chances are that you would be fairly close to the actual temperature. But here are some problems: Suppose you take the poor cricket and toss it into an oven at 450 degrees Fahrenheit, what happens? Well, you are going to hear a large “pop” instead of the predicted 150 chirps per minute! Conversely, toss it into the freezer at -32 degrees Fahrenheit and you will not hear the negative chirps that were predicted in your model. That is the problem of out-of-sample or out-of-range forecasts.



Suppose that in the past, your company spent different amounts in marketing each year and saw improvements in sales and profits as a result of these marketing campaigns. Further assume that, historically, the firm spends between \$10 million and \$20 million in marketing each year, and for every dollar spent in marketing, you get five dollars back in net profits. Does that mean the CEO should come up with a plan to spend \$500 million in marketing the next year? After all, the prediction model says there is a 5x return, meaning the firm will get \$2.5 billion in net profit increase. Clearly this is not going to be the case. If it were, why not keep spending infinitely? The issue here is, again, an out-of-range forecast as well as *nonlinearity*. Revenues will not increase linearly at a multiple of five for each dollar spent in marketing expense, going on infinitely. Perhaps there might be some initial linear relationship, but this will most probably become nonlinear, perhaps taking the shape of a logistic S-curve, with a high-growth early phase followed by some diminishing marginal returns and eventual saturation and decline. After all, how many iPhones can a person own? At some point you have reached your total market potential and any additional marketing you spend will further flood the media airwaves and eventually cut into and reduce your profits. This is the issue of *interactions* among variables.

Think of this another way. Suppose you are a psychologist and are interested in student aptitude in writing essays under pressure. So you round up 100 volunteers, give them a pretest to determine their IQ levels, and divide the students into two groups: the brilliant Group A and the not-so-brilliant Group B, without telling the students, of course. Then you administer a written essay test twice to both groups; the first test has a 30-minute deadline and the second test, with a different but comparably difficult question, a 60-minute window. You then determine if time and intelligence has an effect on exam scores. A well thought out experiment, or so you think. The results might differ depending on whether you gave the students the 30-minute test first and then the 60-minute test or vice versa. As the not-so-brilliant students will tend to be anxious during an exam, taking the 30-minute test first may increase their stress level, possibly causing them to give up easily. Conversely, taking the longer 60-minute test first might make them ambivalent and not really care about doing it well. Of course, we can come up with many other issues with this



experiment. The point is, there might be some interaction among the sequence of exams taken, intelligence, and how students fare under pressure, and so forth.

The student volunteers are just that, volunteers, and so there might be a *self-selection bias*. Another example of self-selection is a clinical research program on sports-enhancement techniques that might only attract die-hard sports enthusiasts, whereas the couch potatoes among us will not even bother participating, let alone be in the mediagraphics readership of the sports magazines in which the advertisements were placed. Therefore, the sample might be biased even before the experiment ever started. Getting back to the student test-taking volunteers, there is also an issue of *survivorship bias*, where the really not-so-brilliant students just never show up for the essay test, possibly because of their negative affinity towards exams. This fickle-mindedness and many other variables that are not *controlled* for in the experiment may actually be reflected in the exam grade. What about the students' facility with English or whatever language the exam was administered in? How about the number of beers they had the night before (being hung over while taking an exam does not help your grades at all)? These are all *omitted variables*, which means that the predictability of the model is reduced should these variables not be accounted for. It is like trying to predict the company's revenues the next few years without accounting for the price increase you expect, the recession the country is heading into, or the introduction of a new, revolutionary product line.

However, sometimes too much data can actually be bad. Now, let us go back to the students again. Suppose you undertake another research project and sample another 100 students, obtain their grade point average at the university, and ask them how many parties they go to on average per week, the number of hours they study on average per week, the number of beers they have per week (the drink of choice for college students), and the number of dates they go on per week. The idea is to see which variable, if any, affects a student's grade on average. A reasonable experiment, or so you think... The issue in this case is *redundant variables* and, perhaps worse, severe *multicollinearity*. In other words, chances are, the more parties they attend, the more people they meet, the more dates they go on per week, and the more drinks they would have on the dates and at the parties, and being



drunk half the time, the less time they have to study! All variables in this case are highly correlated to each other. In fact, you probably only need one variable, such as hours of study per week, to determine the average student's grade point. Adding in all these exogenous variables confounds the forecast equation, making the forecast less reliable.

In fact, when you have severe multicollinearity, which just means there are multiple variables (“multi”) that are changing together (“co-”) in a linear fashion (“linearity”), the regression equation cannot be run. In less severe multicollinearity such as with redundant variables, the adjusted R-square might be high but the p -values will be high as well, indicating that you have a *bad-fitting* model. The prediction errors will be large. While it might be counterintuitive, the problem of multicollinearity, of having too much data, is worse than having less data or having omitted variables. And speaking of bad-fitting models, what is a good R-square *goodness-of-fit* value? This, too, is subjective. How good is your prediction model, and how accurate is it? Unless you measure accuracy using some statistical procedures for your *error measurements* such as those provided by Risk Simulator (e.g., mean absolute deviation, root mean square, p -values, Akaike and Schwartz criterion, and many others) and perhaps input a distributional assumption around these errors to run a simulation on the model, your forecasts may be highly inaccurate.

Another issue is *structural breaks*. For example, remember the poor cricket? What happens when you take a hammer and smash it? Well, there goes your prediction model! You just had a structural break. A company filing for bankruptcy will see its stock price plummet and delisted on the stock exchange, a major natural catastrophe or terrorist attack on a city can cause such a break, and so forth. *Structural shifts* are less severe changes, such as a recessionary period, or a company going into new international markets, or a company engaged in a merger and acquisition, and so forth, where the fundamentals are still there but values might be shifted upward or downward.



Sometimes you run into a *causality loop* problem. We know that correlation does not imply causation. Nonetheless, sometimes there is a *Granger causation*, which means that one event causes another but in a specified direction, or sometimes there is a *causality loop*, where you have different variables that loop around and perhaps back into themselves. Examples of loops include systems engineering where changing an event in the system causes some ramifications across other events, which feeds back into itself causing a feedback loop. Here is an example of a causality loop going the wrong way: Suppose you collect information on crime rate statistics for the 50 states in the United States for a specific year, and you run a regression model to predict the crime rate using police expenditures per capita, gross state product, unemployment rate, number of university graduates per year, and so forth. And further suppose you see that police expenditures per capita is highly predictive of crime rate, which, of course, makes sense, and say the relationship is positive, and if you use these criteria as your prediction model (i.e., the dependent variable is crime rate and independent variable is police expenditure), you have just run into a causality loop issue. That is, you are saying that the higher the police expenditure per capita, the higher the crime rate! Well, then, either the cops are corrupt or they are not really good at their jobs! A better approach might be to use the previous year's police expenditure to predict this year's crime rate; that is, using a *lead* or *lag* on the data. So, more crime necessitates a larger police force, which will, in turn, reduce the crime rate, but going from one step to the next takes time and the lags and leads take the time element into account. Back to the marketing problem, if you spend more on marketing now, you may not see a rise in net income for a few months or even years. Effects are not immediate and the time lag is required to better predict the outcomes.

Many time-series data, especially financial and economic data, are *autocorrelated*; that is, the data are correlated to itself in the past. For instance, January's sales revenue for the company is probably related to the previous month's performance, which itself may be related to the month before. If there is *seasonality* in the variable, then perhaps last January's sales are related to the last 12 months, or January of the year before, and so forth. These seasonal cycles are repeatable



and somewhat predictable. You sell more ski tickets in winter than in summer, and, guess what, next winter you will again sell more tickets than next summer, and so forth. In contrast, *cyclical*ity such as the business cycle, the economic cycle, the housing cycle, and so forth, is a lot less predictable. You can use autocorrelations (relationship to your own past) and lags (one variable correlated to another variable lagged a certain number of periods) for predictions involving seasonality, but, at the same time, you would require additional data. Usually, you will need historical data of at least two seasonal cycles in length to even start running a seasonal model with any level of confidence, otherwise you run into a problem of *micronumerosity*, or lack of data. Regardless of the predictive approach used, the issue of *bad data* is always a concern. Either badly coded data or just data from a bad source, incomplete data points, and *data collection errors* are always a problem in any forecast model.

Next, there is the potential for a *specification error* or using the *incorrect econometric model* error. You can run a seasonal model where there are no seasonalities, thus creating a specification problem, or use an ARIMA when you should be using a GARCH model, creating an econometric model error. Sometimes there are variables that are considered *nonstationary*; that is, the data are not well behaved. These types of variables are really not predictable. An example is stock prices. Try predicting stock prices and you quickly find out that you cannot do a reasonable job at all. Stock prices usually follow something called a random walk, where values are randomly changing all over the place. The mathematical relationship of this random walk is known and is called a *stochastic process*, but the exact outcome is not known for certain. Typically, simulations are required to run random walks, and these stochastic processes come in a variety of forms, including the Brownian motion (e.g., ideal for stock prices), mean-reversion (e.g., ideal for interest rates and inflation), jump-diffusion (e.g., ideal for price of oil and price of electricity), and mixed processes of several forms combined into one. In this case, picking the wrong process is also a specification error.

In most forecasting methods, we assume that the forecast errors are *spherical* or *normally distributed*. That is, the forecast model is the best-fitting model one can develop that minimizes all the forecast errors, which means whatever errors



that are left over are random white noise that is normally distributed (a normal distribution is symmetrical, which means you are equally likely to be underestimating as you are overestimating the forecast). If the errors are not normal and skewed, you are either overestimating or underestimating things, and adjustments need to be made. Further, these errors, because they are random, should be random over time, which means that they should be *identically and independently distributed as normal*, or *i.i.d. normal*. If they are not, then you have some autocorrelations in the data and should be building an autocorrelation model instead.

Finally, if the errors are i.i.d. normal, then the data are *homoskedastic*; that is, the forecast errors are identical over time. Think of it as a tube that contains all your data, and you put a skinny stick in that tube. The amount of wiggle room for that stick is the error of your forecast (and, by extension, if your data is spread out, the tube's diameter is large and the wiggle room is large, which means that the error is large; conversely, if the diameter of the tube is small, the error is small, such that if the diameter of the tube is exactly the size of the stick, the prediction error is zero and your R-squared goodness-of-fit is 100 percent). The amount of wiggle room is constant going into the future. This condition is ideal and what you want. The problem is, especially in nonstationary data or data with some *outliers*, that there is *heteroskedasticity*, which means that instead of a constant diameter tube, you now have a cone, with a small diameter initially that increases over time. This fanning out (see Figure A.19) means that there is an increase in wiggle room or errors the further out you go in time. An example of this fanning out is stock prices, where if the stock price today is \$50, you can forecast and say that there is a 90 percent probability the stock price will be between \$48 and \$52 tomorrow, or between \$45 and \$55 in a week, and perhaps between \$20 and \$100 in six months, holding everything else constant. In other words, the prediction errors increase over time.

So you see, there are many potential issues in forecasting. Knowing your variables and the theory behind the behavior of these variables is an art that depends a lot on experience, comparables with other similar variables, historical data, and expertise in modeling. There is no such thing as a single model that will solve all these issues automatically.



Other than being good modeling practice to create scatter plots prior to performing regression analysis, the scatter plot can also sometimes, on a fundamental basis, provide significant amounts of information regarding the behavior of the data series. Blatant violations of the regression assumptions can be spotted easily and effortlessly, without the need for more detailed and fancy econometric specification tests. For instance, Figure A.12 shows the existence of outliers. Figure A.13's regression results, which include the outliers, indicate that the coefficient of determination is only 0.252 as compared to 0.447 in Figure A.14 when the outliers are removed.

Values may not be identically distributed because of the presence of outliers. Outliers are anomalous values in the data. Outliers may have a strong influence over the fitted slope and intercept, giving a poor fit to the bulk of the data points. Outliers tend to increase the estimate of residual variance, lowering the chance of rejecting the null hypothesis. They may be due to recording errors, which may be correctable, or they may be due to the dependent-variable values not all being sampled from the same population. Apparent outliers may also be due to the dependent-variable values being from the same, but nonnormal, population. Outliers may show up clearly in an X-Y scatter plot of the data, as points that do not lie near the general linear trend of the data. A point may be an unusual value in either an independent or dependent variable without necessarily being an outlier in the scatter plot.

The method of least squares involves minimizing the sum of the squared vertical distances between each data point and the fitted line. Because of this, the fitted line can be highly sensitive to outliers. In other words, least squares regression is not resistant to outliers, thus, neither is the fitted-slope estimate. A point vertically removed from the other points can cause the fitted line to pass close to it, instead of following the general linear trend of the rest of the data, especially if the point is relatively far horizontally from the center of the data (the point represented by the mean of the independent variable and the mean of the dependent variable). Such points are said to have high leverage: the center acts as a fulcrum, and the fitted line pivots toward high-leverage points, perhaps fitting the main body of the data poorly. A data point that is extreme in dependent variables but lies near the center of the



data horizontally will not have much effect on the fitted slope, but by changing the estimate of the mean of the dependent variable, it may affect the fitted estimate of the intercept.



Figure A.12. Scatter Plot Showing Outliers

Regression Statistics	
R-Squared (Coefficient of Determination)	0.2520
Adjusted R-Squared	0.2367
Multiple R (Multiple Correlation Coefficient)	0.5020
Standard Error of the Estimates (SEy)	3417.60

Regression Results		
	Intercept	Marketing
Coefficients	53.2690	0.4857
Standard Error	11.6769	0.1207
t-Statistic	4.5619	4.0247
p-Value	0.0000	0.0002

Figure A.13. Regression Results with Outliers



Regression Statistics		
R-Squared (Coefficient of Determination)	0.4470	HIGHER R-SQUARED
Adjusted R-Squared	0.4343	WHEN OUTLIERS
Multiple R (Multiple Correlation Coefficient)	0.6686	ARE REMOVED
Standard Error of the Estimates (SEy)	2524.90	

Regression Results		
	Intercept	Marketing
Coefficients	19.4470	0.8229
Standard Error	13.4006	0.1365
t-Statistic	1.4512	6.0267
p-Value	0.1532	0.0000

Figure A.14. Regression Results with Outliers Deleted

However, great care should be taken when deciding if the outliers should be removed. Although in most cases when outliers are removed, the regression results look better, *a priori* justification must first exist. For instance, if one is regressing the performance of a particular firm's stock returns, outliers caused by downturns in the stock market should be included; these are not truly outliers as they are inevitabilities in the business cycle. Forgoing these outliers and using the regression equation to forecast one's retirement fund based on the firm's stocks will yield incorrect results at best. In contrast, suppose the outliers are caused by a single nonrecurring business condition (e.g., merger and acquisition) and such business structural changes are not forecast to recur; then these outliers should be removed and the data cleansed prior to running a regression analysis.

Figure A.15 shows a scatter plot with a nonlinear relationship between the dependent and independent variables. In a situation such as this one, a linear regression will not be optimal. A nonlinear transformation should first be applied to



the data before running a regression. One simple approach is to take the natural logarithm of the independent variable (other approaches include taking the square root or raising the independent variable to the second or third power) and regress the sales revenue on this transformed marketing-cost data series. Figure A.16 shows the regression results with a coefficient of determination at 0.938, as compared to 0.707 in Figure A.17 when a simple linear regression is applied to the original data series without the nonlinear transformation.

If the linear model is not the correct one for the data, then the slope and intercept estimates and the fitted values from the linear regression will be biased, and the fitted slope and intercept estimates will not be meaningful. Over a restricted range of independent or dependent variables, nonlinear models may be well approximated by linear models (this is, in fact, the basis of linear interpolation), but for accurate prediction, a model appropriate to the data should be selected. An examination of the X-Y scatter plot may reveal whether the linear model is appropriate. If there is a great deal of variation in the dependent variable, it may be difficult to decide what the appropriate model is; in this case, the linear model may do as well as any other, and has the virtue of simplicity.

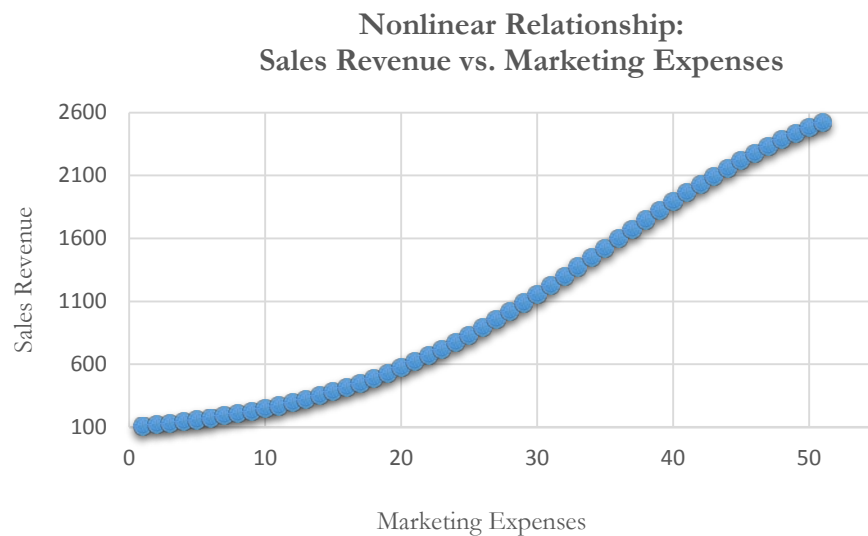


Figure A.15. Scatter Plot Showing a Nonlinear Relationship

Regression Statistics		
R-Squared (Coefficient of Determination)	0.9380	MUCH HIGHER
Adjusted R-Squared	0.9364	SIGNIFICANCE
Multiple R (Multiple Correlation Coefficient)	0.9685	WITH NONLINEAR
Standard Error of the Estimates (SEy)	101.74	TRANSFORMATION

Regression Results		
	Intercept	LN(Marketing)
Coefficients	10.2080	5.3783
Standard Error	1.0618	0.2001
t-Statistic	9.6141	26.8750
<i>p</i> -Value	0.0000	0.0000

Figure A.16. Regression Results Using a Nonlinear Transformation

However, great care should be taken here as the original linear data series of marketing costs should not be added with the nonlinearly transformed marketing costs in the regression analysis. Otherwise, multicollinearity occurs; that is, marketing costs are highly correlated to the natural logarithm of marketing costs, and if both are used as independent variables in a multivariate regression analysis, the assumption of no multicollinearity is violated and the regression analysis breaks down. Figure A.18 illustrates what happens when multicollinearity strikes. Notice that the coefficient of determination (0.938) is the same as the nonlinear transformed regression (Figure A.16). However, the adjusted coefficient of determination went down from 0.9364 (Figure A.16) to 0.9358 (Figure A.18). In addition, the previously statistically significant marketing-costs variable in Figure A.17 now becomes insignificant (Figure A.18) with a probability or *p*-value increasing from close to zero



to 0.4661. A basic symptom of multicollinearity is low t-statistics coupled with a high R-squared (Figure A.18).

Regression Statistics		
R-Squared (Coefficient of Determination)	0.7070	LINEAR REGRESSION
Adjusted R-Squared	0.7013	RETURNS LOWER
Multiple R (Multiple Correlation Coefficient)	0.8408	R-SQUARED THAN
Standard Error of the Estimates (SEy)	477.72	NONLINEAR MODEL

Regression Results		
	Intercept	Marketing
Coefficients	33.3580	0.0164
Standard Error	0.6335	0.0015
t-Statistic	52.6580	10.7720
p-Value	0.0000	0.0000

Figure A.17. Regression Results Using Linear Data



Regression Statistics		
R-Squared (Coefficient of Determination)	0.9380	WATCH OUT FOR
Adjusted R-Squared	0.9358	MULTICOLLINEARITY
Multiple R (Multiple Correlation Coefficient)	0.9685	
Standard Error of the Estimates (SEy)	100.59	

Regression Results			
	Intercept	Marketing	LN(Marketing)
Coefficients	9.0966	-0.0011	5.6542
Standard Error	1.8510	0.0015	0.4606
t-Statistic	4.9143	-0.7349	12.2750
p-Value	0.0000	0.4660	0.0000

NOTE THAT NONLINEAR OVERTAKES LINEAR MODEL... A SYMPTOM THAT MULTICOLLINEARITY MAY EXIST: LOW P-VALUE AND HIGH R-SQUARED

Figure A.18. Regression Results Using Both Linear and Nonlinear Transformations

Another common violation is heteroskedasticity, that is, the variance of the errors increases over time. Figure A.19 illustrates this case, where the width of the vertical data fluctuations increases or fans out over time. In this example, the data points have been changed to exaggerate the effect. However, in most time-series analysis, checking for heteroskedasticity is a much more difficult task. And correcting for heteroskedasticity is an even greater challenge.³⁹ Notice in Figure A.20 that the coefficient of determination dropped significantly when heteroskedasticity exists. As is, the regression model is insufficient and incomplete.

If the variance of the dependent variable is not constant, then the error's variance will not be constant. The most common form of such heteroskedasticity in



the dependent variable is that the variance of the dependent variable may increase as the mean of the dependent variable increases for data with positive independent and dependent variables.

Unless the heteroskedasticity of the dependent variable is pronounced, its effect will not be severe: the least-squares estimates will still be unbiased, and the estimates of the slope and intercept will either be normally distributed if the errors are normally distributed, or at least normally distributed asymptotically (as the number of data points becomes large) if the errors are not normally distributed. The estimate for the variance of the slope and overall variance will be inaccurate, but the inaccuracy is not likely to be substantial if the independent-variable values are symmetric about their mean.

Heteroskedasticity of the dependent variable is usually detected informally by examining the X-Y scatter plot of the data before performing the regression. If both nonlinearity and unequal variances are present, employing a transformation of the dependent variable may have the effect of simultaneously improving the linearity and promoting equality of the variances. Otherwise, a weighted least-squares linear regression may be the preferred method of dealing with nonconstant variance of the dependent variable.



Figure A.19. Scatter Plot Showing Heteroskedasticity With Nonconstant Variance



Regression Statistics		
R-Squared (Coefficient of Determination)	0.3980	WATCH OUT FOR
Adjusted R-Squared	0.3858	HETEROSKEDASTICITY!
Multiple R (Multiple Correlation Coefficient)	0.6309	

Regression Results		
	Intercept	Marketing
Coefficients	1.5742	0.9586
Standard Error	16.7113	0.1701
t-Statistic	0.0942	5.6371
p-Value	0.9253	0.0000

Figure A.20. Regression Results With Heteroskedasticity

Other Technical Issues in Regression Analysis

If the data to be analyzed by linear regression violate one or more of the linear regression assumptions, the results of the analysis may be incorrect or misleading. For example, if the assumption of independence is violated, then linear regression is not appropriate. If the assumption of normality is violated or outliers are present, then the linear regression goodness-of-fit test may not be the most powerful or informative test available, and this could mean the difference between detecting a linear fit or not. A nonparametric, robust, or resistant regression method, a transformation, a weighted least-squares linear regression, or a nonlinear model may result in a better fit. If the population variance for the dependent variable is not constant, a weighted least-squares linear regression or a transformation of the dependent variable may provide a means of fitting a regression adjusted for the inequality of the variances. Often, the impact of an assumption violation on the linear regression result depends on the extent of the violation (such as how nonconstant



the variance of the dependent variable is, or how skewed the dependent variable population distribution is). Some small violations may have little practical effect on the analysis, while other violations may render the linear regression result useless and incorrect.

Other potential assumption violations include

- lack of independence in the dependent variable;
- independent variable is random, not fixed;
- special problems with few data points; and
- special problems with regression through the origin.

Lack of Independence in the Dependent Variable

Whether the independent-variable values are independent of each other is generally determined by the structure of the experiment from which they arise. The dependent-variable values collected over time may be autocorrelated. For serially correlated dependent-variable values, the estimates of the slope and intercept will be unbiased, but the estimates of their variances will not be reliable and, hence, the validity of certain statistical goodness-of-fit tests will be flawed. An ARIMA model may be better in such circumstances.

The Independent Variable Is Random, Not Fixed

The usual linear regression model assumes that the observed independent variables are fixed, not random. If the independent values are not under the control of the experimenter (i.e., are observed but not set), and if there is in fact underlying variance in the independent variable, but the variance is the same, the linear model is called an errors-in-variables model or a structural model. The least-squares fit will still give the best linear predictor of the dependent variable, but the estimates of the slope and intercept will be biased (will not have expected values equal to the true slope and variance). A stochastic forecast model may be a better alternative here.



Special Problems With Few Data Points or Micronumerosity

If the number of data points is small (also termed *micronumerosity*), it may be difficult to detect assumption violations. With small samples, assumption violations such as nonnormality or heteroskedasticity of variances are difficult to detect even when they are present. With a small number of data points, linear regression offers less protection against violation of assumptions. With few data points, it may be hard to determine how well the fitted line matches the data, or whether a nonlinear function would be more appropriate.

Even if none of the test assumptions are violated, a linear regression on a small number of data points may not have sufficient power to detect a significant difference between the slope and zero, even if the slope is nonzero. The power depends on the residual error, the observed variation in the independent variable, the selected significance alpha level of the test, and the number of data points. Power decreases as the residual variance increases, decreases as the significance level is decreased (i.e., as the test is made more stringent), increases as the variation in observed independent variable increases, and increases as the number of data points increases. If a statistical significance test with a small number of data points produces a surprisingly nonsignificant probability value, then lack of power may be the reason. The best time to avoid such problems is in the design stage of an experiment, when appropriate minimum sample sizes can be determined, perhaps in consultation with an econometrician, before data collection begins.

Special Problems With Regression Through the Origin

The effects of nonconstant variance of the dependent variable can be particularly severe for a linear regression when the line is forced through the origin: the estimate of variance for the fitted slope may be much smaller than the actual variance, making the test for the slope nonconservative (more likely to reject the null hypothesis that the slope is zero than what the stated significance level indicates). In general, unless there is a structural or theoretical reason to assume that the intercept is zero, it is preferable to fit both the slope and intercept.



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Biographies

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EXPERIENCE

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- Associate Professor of Information and Operations Management, University of Southern California, Marshall School of Business, 8/97–8/01.
- Visiting Professor of Information and Operations Management, University of Southern California, Marshall School of Business, 8/95–8/97.
- Academic Program Director, University of Southern California, Marshall School of Business, 8/98–8/01.
- Director (Vice President), Consumer Behavior Research in Telematics and Informatics, Centro Studies. Salvador (Telecom Italia), 1/94–8/95.
- Chief Business Process Engineer, Strategic Information Systems Division, Pacific Bell, 10/92–1/94.
- Director, Business Development and Domain Engineering, Strategic Information Systems Division, Pacific Bell, 8/91–10/92.
- Assistant Professor of Business Communication (clinical), University of Southern California, School of Business Administration, 1982–1991.



- Associate Director, Center for Operations Management Education and Research, University of Southern California, 1987–1991.
- Associate Director, Center for Telecommunications Management, University of Southern California, 1984–1985.
- Assistant Professor, University of Kentucky, College of Communications, 1977–1982.

PUBLICATIONS

- Rodgers, W., & Housel, T. (2009). Problems and resolutions to future knowledge-based assets reporting. *Journal of Intellectual Capital*, 10(4).
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AWARDS

Honorable Mention 1994 Planning Forum Case Competition

First Prize Winner 1986 Society for Information Management paper competition

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