Spacecraft Vibration Reduction Using Pulse-Width Pulse-Frequency Modulated Input Shaper

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Minimizing vibrations of a flexible spacecraft actuated by on-off thrusters is a challenging task. This paper presents the first study of pulse-width pulse-frequency modulated thruster control using command input shaping. Input shaping is a technique that uses a shaped command to ensure zero residual vibration of a flexible structure. Pulse-width pulse-frequency modulation is a control method that provides pseudolinear operation for an on-off thruster. The proposed method takes full advantage of the pseudolinear property of a pulse-width pulse-frequency modulator and integrates it with a command shaper to minimize the vibration of a flexible spacecraft induced by on-off thruster firing. Compared to other methods, this new approach has numerous advantages: 1) effectiveness in vibration suppression, 2) dependence only on modal frequency and damping, 3) robustness to variations in modal frequency and damping, and 4) easy computation. Numerical simulations performed on an eight-mode model of the Flexible Spacecraft Simulator in the Spacecraft Research and Design Center at the U.S. Naval Postgraduate School demonstrate the efficacy and robustness of the method.

I. Introduction

Most modern spacecraft attitude control systems employ on-off thrusters. On-off thrusters produce discontinuous and nonlinear control actions. These control actions may excite flexible modes of modern spacecraft, which use large, complex, and lightweight structures such as solar array panels. Designing an...
on-off thruster control system to provide fine pointing accuracy while avoiding interaction with the flexible structure poses a challenging task.

Research toward this end has been focused mainly into two areas. In one area, efficient methods to convert continuous input commands to on-off signals suitable for controlling on-off thrusters are sought. The other area focuses on modifying an existing command so that it results in less or zero residual vibration of a flexible spacecraft.

The two major approaches for thruster control are bang-bang and pulse modulation. Bang-bang control is simple in formulation, but results in excessive thruster action. Its discontinuous control actions often interact with the flexible mode of the spacecraft and result in limit cycles. Therefore, bang-bang control is not commonly used. On the other hand, pulse modulators are commonly employed due to their advantages of reduced propellant consumption and near-linear duty cycle. In general, pulse modulators produce a pulse command sequence to the thruster valves by adjusting pulse width and/or pulse frequency. Pulse modulators such as pseudorate modulator, integral-pulse frequency modulator, and pulse-width and pulse-frequency (PWPF) modulator have been proposed. Among these, the PWPF modulator holds several superior advantages such as close to linear range the average torque produced equals the demand torque input. Compared with other methods of modulation, the

| Table 1 FSS cantilever and system frequencies |
|-----------------|----------------|----------------|----------------|
| Mode | Cantilever Hz | System Hz | |
| 1 | 0.183 | 0.213 | 1.34 |
| 2 | 0.452 | 0.504 | 3.16 |
| 3 | 2.41 | 2.42 | 15.23 |
| 4 | 4.23 | 4.25 | 26.72 |
| 5 | 8.42 | 8.42 | 52.94 |
| 6 | 12.3 | 12.3 | 77.31 |
| 7 | 16.6 | 16.6 | 104.2 |
| 8 | 21.0 | 21.0 | 132.1 |

where $\theta$ is the angular position of the main body, $q_i$ is the modal coordinate for the $i$th cantilever mode, $I_z$ is the moment of inertia of the system, $D_i$ is the rigid-elastic coupling for the $i$th mode, $T_c$ is the control torque, $T_d$ is the disturbance torque, $\zeta$ is the damping ratio of the $i$th mode, and $\omega_n$ is the natural frequency for the $i$th mode. The rigid-elastic coupling $D_i$ is given by

$$D_i = \int \left( x_F \phi_i^y - y_F \phi_i^x \right) d\tau$$

where $x_F$ and $y_F$ are the coordinates of a point on the flexible structure, and $\phi_i^x$ and $\phi_i^y$ are respectively the $x$ and $y$ component of the $i$th modal vector at that point.

In discretizing the system via the finite element method, the number of modes was truncated at eight to obtain a compromise between reasonable model accuracy and computational feasibility.

The model is placed into state space in preparation for digital simulation using the MATLAB Simulink software package. The state-space representation of the system equations is

$$\dot{x} = Ax + Bu$$

where the state vector $x$ is defined as

$$x = [\theta, \dot{\theta}, q_1, \ldots, q_8, \dot{q}_1, \ldots, \dot{q}_8]^T$$

The output $y$ is the vector of the states, hence, $C$ is an $18 \times 18$ identity matrix, and the feedback values of angular position and rate are measured exactly.

To find the natural frequencies of the flexible-appendage and rigid-body system, a MATLAB routine is used. The cantilever and system frequencies are listed in Table 1. Because low-frequency modes are generally dominant in a flexible system, in this paper the goal of design is to suppress the low-frequency-mode vibrations.

II. Flexible Spacecraft Simulator

The U.S. Naval Postgraduate School’s FSS simulates motion about the pitch axis of a spacecraft. As shown in Fig. 1, the FSS is composed of a rigid central body and a reflector supported by an “L”-shape flexible appendage. The center body represents the main body of the spacecraft, whereas the flexible appendage represents a flexible antenna support structure. The flexible appendage is composed of a base beam cantilevered to the main body and a tip beam connected to the base beam at a right angle with a rigid elbow joint. The flexible appendage is supported by one air pad each at the elbow and tip to minimize friction.

The flexible dynamic model used in this study is derived using the hybrid-coordinate formulation. The equations describing the motion of the rigid-body and the flexible appendage are

$$\begin{align*}
I_c \ddot{\theta} + \sum_{i=1}^8 D_i \ddot{q}_i &= T_c + T_d \\
\ddot{q}_i + 2\zeta_m \omega_n \dot{q}_i + \omega_n^2 q_i + D_i \dot{\theta} &= 0
\end{align*}$$

where $\theta$ is the angular position of the main body, $q_i$ is the modal coordinate for the $i$th cantilever mode, $I_c$ is the moment of inertia of the system, $D_i$ is the rigid-elastic coupling for the $i$th mode, $T_c$ is the control torque, $T_d$ is the disturbance torque, $\zeta_m$ is the damping ratio of the $i$th mode, and $\omega_n$ is the natural frequency for the $i$th mode.

III. PWPF Modulator

The PWPF modulator produces a pulse command sequence to the thruster valves by adjusting the pulse width and pulse frequency. In its linear range the average torque produced equals the demand torque input. Compared with other methods of modulation, the
PWPF modulator has several superior advantages such as close-to-linear operation, high accuracy, and adjustable PWPF, which provide scope for advanced control.

As shown in Fig. 2, the PWPF modulator is composed of a Schmidt Trigger, a prefilter, and a feedback loop. A Schmidt Trigger is simply an on-off relay with a deadband and hysteresis. When a positive input to the Schmidt Trigger is greater than a threshold (also denoted as \( E_{on} \)), the trigger output is \( U_{on} \). Consequently, when the input falls below \( d - h \) (also denoted as \( E_{off} \)), the trigger output is 0. This response is also reflected for negative inputs. The error signal \( e(t) \) is the difference between the Schmidt Trigger output \( U_{on} \) and the system input \( r(t) \). The error is fed into the prefilter whose output \( f(t) \) feeds the Schmidt Trigger. The parameters of interest are the prefilter coefficients \( k_{m} \) and \( \tau_{m} \), the input gain \( K_p \), and the Schmidt Trigger parameters \( d \) and \( h \).

### A. Modulator Static Characteristics

With a constant input the modulator has a behavior that is independent of the system in which it is used. The pulse width and period are usually fast compared with the system dynamics, and so the input to the modulator (the error signal feedback) changes slowly; the static characteristics are a good indication of how the modulator will work in most cases. Choosing appropriate parameters so that the modulator has desired static characteristics is the first step of attitude control design using the PWPF modulator.

#### 1. On-Time and Off-Time

If the input \( e(t) \) to the prefilter is a constant, e.g., \( e_0 \), then the relationship between \( f(t) \) and \( e_0 \) can be represented by

\[
f(t) = f(0) + [k_m e_0 - f(0)](1 - e^{-t/\tau_m})
\]  

From Eq. (3) we have, as \( t \to \infty \),

\[
f(t \to \infty) = k_m e_0
\]

The time taking the prefilter output to transit from \( d \) to \( d - h \) is defined as the relay on-time or pulse width, denoted by \( T_{on} \) or \( PW \). \( T_{on} \) or \( PW \) can be solved from Eq. (3) by setting \( f(0) = d \), \( f(T_{on}) = d - h \), and \( t = T_{on} \):

\[
T_{on} = PW = -\tau_m \ln \left[ 1 + \frac{h}{k_m [r(t) - U_{on}] - d} \right]
\]

The off-time is defined as the time taking the prefilter output from \( 0 \) to \( d \). According to Eq. (3), the off-time denoted by \( T_{off} \) can be solved by setting \( f(0) = 0 \), \( f(T_{off}) = d \), and \( t = T_{off} \):

\[
T_{off} = -\tau_m \ln \left[ 1 - \frac{h}{k_m r(t) - (d - h)} \right]
\]

#### 2. Modulator Frequency

The frequency of the PWPF modulator is defined as the inverse of the period of the PWPF cycle and is given by the following equation:

\[
f = \frac{1}{T_{on} + T_{off}}
\]

#### 3. Modulation Factor

The modulation factor of the PWPF controller is the ratio of the relay on-time to the period and is given by

\[
MF = \frac{T_{on}}{T_{on} + T_{off}}
\]

#### 4. Conditions for Pseudolinear Operation

The maximum input \( r_{max} \) for pseudolinear operation can be solved by equating the maximum value of the prefilter output \( k_m (r_{max} - U_{on}) \) to the Schmidt Trigger off condition \( d - h \):

\[
k_m (r_{max} - U_{on}) = d - h
\]

i.e.,

\[
k_m U_{on} - d = 1
\]

or

\[
r_{max} = U_{on} + (d - h)/k_m
\]

The effective deadband of the modulator is defined as the minimum input to the modulator so that \( T_{on} > 0 \). The \( r_{min} \) can be determined by equating the prefilter output when the Schmidt Trigger output is zero to the Schmidt Trigger on condition,

\[
k_m r_{min} = d
\]

i.e.,

\[
r_{min} = d/k_m
\]

It is clear that an increase of \( k_m \) reduces the size of deadband. It is reasonable to keep \( k_m > 1 \) to ensure that the on-threshold \( d \) is an upper bound on the deadband. With the use of the input gain \( K_p \), the effective deadband is

\[
r_{min} = d/k_m K_p
\]

The net effect of input gain \( K_p \) is altering the deadband by scaling the input signal. When an input signal has a large amplitude and does not fall inside the deadband, a small \( K_p \) should be used to reduce thruster activity. On the other hand, when the input signal has a small amplitude and falls inside the deadband, a large \( K_p \) is required to force the input out of the deadband. In this case, a large \( K_p \) can maintain linearity of the modulator and increase control accuracy. Using appropriate \( K_p \) according to the magnitude of input signal is an effective way to maintain modulator linearity and reduce thruster activity. In this paper, a two-staged input gain \( K_p \) will be used, as to be discussed in a later section.

#### 5. Minimum Pulse Width Determination

The effective deadband of the modulator is defined as the minimum input to the modulator for which \( T_{on} > 0 \). Substituting Eq. (9) into Eq. (6) gives an expression for the minimum on-time, defined as the minimum pulse width. The minimum pulse width is usually dictated by relay operational constraints and is given by

\[
T_{min} = -\tau_m \ln [1 - h/(k_m U_{on})]
\]
B. PWPF Modulator Design Analysis

The objective of this analysis is to recommend appropriate PWPF modulator parameter settings for general use. The recommended settings will be used later for design of a PWPF modulator to modulate the command shaped by an input shaper. The design analysis is done by comparing performance indices for different modulator parameter settings with the help of MATLAB/Simulink.

1. Static Analysis

Simulations with a constant input $r = 0.5$ are performed to study the impact of parameters $\{E_{\text{on}}(d), E_{\text{off}}(d-h), k_m, K_p\}$ on the PWPF static performance indices: modulation factor, thruster firing frequency, thruster cycles, and total thruster on-time. Figure 3 plots the number of thruster firings vs $E_{\text{on}}$ and $E_{\text{off}}/E_{\text{on}}$. This figure indicates that for $E_{\text{off}}/E_{\text{on}} > 0.8$ the thruster cycle increases much faster than it does for $E_{\text{off}}/E_{\text{on}} < 0.2$. Figure 3 shows that for $E_{\text{on}} < 0.2$ the thruster cycle increases much faster than it does for $E_{\text{on}} > 0.2$. To avoid excessive thruster firings, $E_{\text{on}} > 0.2$ and $E_{\text{off}}/E_{\text{on}} < 0.8$ are suggested in PWPF modulator designs. Simulations with varying $k_m$ and $K_p$ show that keeping $1 < k_m < 6.0$ and $2 < K_p < 10$ can maintain pseudolinear operation of a PWPF modulator without excessive thruster firings. The preferred range of parameters is recommended in Table 2.

2. Dynamic Analysis

To study the impact of input frequency and the time constant on PWPF output phase lag and thruster activity, simulations are conducted by applying unity magnitude sinusoidal inputs to the PWPF modulator. Input frequencies are varied from 1 to 150 rad/s, and time constants are varied from 0.01 to 0.4s. Fixed modulator parameters are shown in Table 3 and are consistent with the recommendations in Table 2. The input gain is set to one.

Phase Lag. The result of the phase lag simulation is shown in Fig. 4. The value of phase lag, displayed on the vertical axis, is represented in terms of the percentage of a period of the input signal. For example, zero on the vertical scale indicates no phase lag. A value of 0.5 indicates a phase lag of 50% of an input period.

Note that for $\tau_m$ less than 0.2, there is little phase lag for all input frequencies. The plateau shown by a phase lag of 400% indicates the region of zero modulation factor. In this area the time constant is too large for the modulator to react to the high-frequency input. Note that for $\tau_m$ greater than approximately 0.2, the phase lag increases monotonically at low frequency. These characteristics suggest that $\tau_m$ should be kept less than 0.2 to reduce phase lag.

Thruster Activity. Figure 5 shows the effect of time constant and input frequency on thruster cycles. This graph suggests that a minimum time constant value of 0.1 should be maintained to avoid frequent thruster firings (Fig. 4). Further simulations also show that maintaining $\tau_m$ greater than 0.1 avoids excessive propellant use.

3. Fourier Transform Analysis

To better understand PWPF modulation, Fourier transform of the output of a PWPF modulator in pseudolinear operation is performed.

### Table 2 Static analysis results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Recommended setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulator gain, $k_m$</td>
<td>$1 &lt; k_m &lt; 6.0$</td>
</tr>
<tr>
<td>On-threshold, $E_{\text{on}}(d)$</td>
<td>$&gt;0.2$</td>
</tr>
<tr>
<td>Off-threshold, $E_{\text{off}}(d-h)$</td>
<td>$&lt;0.8d$</td>
</tr>
<tr>
<td>Input gain, $K_p$</td>
<td>$2 &lt; K_p &lt; 10$</td>
</tr>
</tbody>
</table>

### Table 3 PWPF parameters in dynamic simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$</td>
<td>4.5</td>
</tr>
<tr>
<td>$E_{\text{on}}(d)$</td>
<td>0.45</td>
</tr>
<tr>
<td>$E_{\text{off}}(d-h)$</td>
<td>0.15</td>
</tr>
<tr>
<td>$U_m$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Fig. 4 Phase lag of PWPF output.
and compared with that of the input sinusoidal signal, as shown in Fig. 6. This figure indicates some minor frequency components in the PWPF output besides the main component (input frequency). These extra frequency components generated by a PWPF modulator must be taken into consideration when the modulator is used to modulate the command of an input shaper.

4. Design Recommendation

Table 4 summarizes the results from Secs. III.A and III.B and shows the recommended setting for each parameter. The PWPF parameter settings recommended in Table 4 can be used not only in this paper but also as general guidelines for PWPF modulator design.

### Table 4 Summary of PWPF design analyses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Static analysis</th>
<th>Dynamic analysis</th>
<th>Recommended settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_m$</td>
<td>1.0 &lt; 6.0</td>
<td>N/A</td>
<td>1.0 &lt; 6.0</td>
</tr>
<tr>
<td>$K_p$</td>
<td>2.0 &lt; 10</td>
<td>N/A</td>
<td>2.0 &lt; 10</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>N/A</td>
<td>0.1–0.2</td>
<td>0.1–0.2</td>
</tr>
<tr>
<td>$E_{on}(d)$</td>
<td></td>
<td></td>
<td>&gt;0.2</td>
</tr>
<tr>
<td>$E_{off}(d-h)$</td>
<td></td>
<td></td>
<td>&lt;0.8$d$</td>
</tr>
</tbody>
</table>

The ZV shaper offers the shortest pulse train that can cancel a single-mode vibration; however, it requires very good knowledge of the plant. Singer and Seering\(^\text{12}\) showed that the ZV shaper was robust for only small variations (±5\%) in modal frequency. To enhance a shaper’s robustness, a zero vibration derivative (ZVD) shaper with three impulses and a zero vibration derivative derivative (ZVDD) shaper with four impulses were developed.\(^\text{12}\) The ZVD shaper provides robustness for up to ±20\% variations in frequency, and the ZVDD shaper allows plant uncertainties on the order of ±40\% while retaining the ZV characteristics. Figure 8 illustrates the impulse train for a ZVDD shaper.

The pulse train parameters are given by

$$K = \exp\left(\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}\right), \quad \Delta T = \frac{\pi}{\omega_0 \sqrt{1 - \zeta^2}}$$

where $\omega_0$ is the undamped modal frequency and $\zeta$ is the damping ratio for this mode.

ZV, ZVD, and ZVDD shapers are designed for systems with proportional actuators, and they cannot be applied directly to systems with on-off actuators. By imposing constraints of constant-amplitude commands and maneuver requirements, ZV, ZVD, and ZVDD shapers are extended to constant amplitude pulse (CAP) shapers.\(^\text{15}\) Obtaining a CAP shaper often involves complicated optimization. CAP shapers also result in bang-bang control action.

With the increase of the number of pulses used in the shaper, the shaped command will result in slower rigid-body response. Therefore, among ZV, ZVD, and ZVDD shapers, ZV has the fastest rigid-body response, and ZVDD has the slowest response.
For multimode flexible systems the shaper aiming at one fundamental mode may excite a higher mode, but overall performance can be improved. Compared with CAP shapers, variable-amplitude shapers like ZV, ZVD, and ZVDD shapers have the advantage of simple computation because ZV, ZVD, and ZVDD shapers do not require optimization.

V. Integrated Input Shaper and PWPF Modulator for Vibration Reduction

A. PWPF Modulator

According to the recommended parameter settings in Table 4, the parameters of the PWPF modulator are chosen and listed in Table 5. This modulator will be used to modulate the commands that have been modified by an input shaper.

A two-staged design of $K_p$ is intended to bring the PWPF modulator prefilter output above the deadzone level. When the input falls inside the deadzone, a large value of $K_p$ is used; otherwise, the recommended minimum value is used.

The PWPF modulator parameter settings in Table 4 are general recommendations. The PWPF modulator parameters used in this case (shown in Table 5) can generally remain the same even when the modulator is used for a different shaper.

B. Input Shaper

As shown in Sec. III.B.3, the PWPF modulator does not exactly replicate the input command frequency. This motivates the use of a ZVDD shaper for increased robustness with respect to modal frequency. Because the goal is to suppress vibrations of low-frequency modes, a four-mode ZVDD shaper is chosen.

The design method presented in Sec. IV is used to generate the pulse trains for the ZVDD shaper. The resulting four-impulse sequence for each mode is given by

$$
\begin{align*}
A_j(t) &= \begin{bmatrix}
1 & 3K_1 & 3K_2^2 & K_3^3 \\
0 & \Delta T & 2\Delta T & 3\Delta T
\end{bmatrix}
\end{align*}
$$

where $K$ and $\Delta T$ are defined in Sec. IV and the sequence is unity normalized by

$$X_{DD} = 1 + 3K + 3K^2 + K^3$$

The resulting ZVDD pulse trains for modes 1-4 of the FSS are

Mode 1:

$$
\begin{align*}
A_j(t) &= \begin{bmatrix}
0.1274 & 0.3773 & 0.3726 & 0.1227 \\
0 & 1.9563 & 3.9127 & 5.8690
\end{bmatrix}
\end{align*}
$$

Mode 2:

$$
\begin{align*}
A_j(t) &= \begin{bmatrix}
0.1274 & 0.3773 & 0.3726 & 0.1227 \\
0 & 0.8273 & 1.6547 & 2.8420
\end{bmatrix}
\end{align*}
$$

Mode 3:

$$
\begin{align*}
A_j(t) &= \begin{bmatrix}
0.1274 & 0.3773 & 0.3726 & 0.1227 \\
0 & 0.1719 & 0.3437 & 0.5156
\end{bmatrix}
\end{align*}
$$

Mode 4:

$$
\begin{align*}
A_j(t) &= \begin{bmatrix}
0.1274 & 0.3773 & 0.3726 & 0.1227 \\
0 & 0.0980 & 0.1960 & 0.2940
\end{bmatrix}
\end{align*}
$$

Note that the amplitudes are the same due to the same damping assumed for all modes. The preceding four impulse trains are convolved to generate the four-mode ZVDD input as shown in Fig. 9. For comparison purposes the unshaped step command is presented in Fig. 9 as well. Using the shaped command, a longer settling time for the rigid-body is expected.

C. Vibration Reduction Using PWPF Modulated Input Shaper

The PWPF modulator proposed in Sec. V.A is used to modulate the four-mode ZVDD shaped command proposed in Sec. V.B. The primary goal is to reduce the lower-mode vibration of the FSS during a slew. As associated with an input shaper, worse performance in higher modes may be expected but should be in a limited range. Simulations are done to analyze the impact of the control with a PWPF modulated input shaper on rigid-body performance and flexible mode responses. The block diagram illustrating the FSS control system is shown in Fig. 10.

Figure 11 shows the lower-mode excitations resulting from a 10-deg slew maneuver. With all modal damping ratios of 0.004, the lower-mode flexible response is essentially undamped for the duration of the simulation when an unshaped step command is used. Using a four-mode ZVDD shaper with the PWPF modulator results in excellent cancellation of the targeted modes. Reductions in modal excitations of up to 95% are achieved in the first two modes.
Fig. 11 FSS slewing with PWPF modulated ZVDD shaper.

Fig. 12 Rigid-body response with ±20% modal frequency uncertainty.

(a) Mode 1
(b) Mode 2

Fig. 13 ZVDD shaper robustness to 20% modal frequency uncertainty.

(a) Mode 1
(b) Mode 2

D. Robustness Analysis

Consider that in practice modal frequency generally can be obtained within ±20% error. The first simulation is run with ±20% errors in all four modal frequencies of the four-mode ZVDD shaper, and the results are compared with that of the case with exactly known modal frequencies. The rigid-body responses are shown in Fig. 12, and the first two mode responses are shown in Fig. 13. Figure 12 reveals that error in modal frequency slightly changes the settling time; however, it has little impact on the final stage error. Figure 13a shows that the case of −20% frequency error is very close to the nominal case, whereas the case of +20% frequency has a slightly increased vibration. Either case is robust with the error in modal frequency. Figure 13b also illustrates that ±20% frequency variations in all four modes have very limited influence on mode 2 vibration. Modes 3 and 4 show the same trend (figures not shown because of space limitation).

To further study the robustness, simulations are run using frequencies varying from $0.2\omega_0$ to $2.0\omega_0$ and damping ratios varying from $0.1\zeta$ to $2.0\zeta$ for all four modes of the four-mode ZVDD shaper. Flexible mode responses in terms of their average absolute displacement are shown in Figs. 14a–14d. Several observations are made here. First, vibration increases caused by ±20% modal frequency error are small for the first four modes (Figs. 14a–14d). Second, Fig. 14 reveals that the ZVDD shaper is almost insensitive to variations in damping. Third, the PWPF modulated shaper achieves well-behaved modal responses even for modal frequency errors of 100%.

The preceding three observations verify the robustness of the proposed vibration reduction method. In summary, integrating the techniques of command input shaping and PWPF modulation combines the advantages of variable amplitude input shapers and PWPF modulators. It provides a simple, effective, and robust method to suppress vibration on flexible spacecraft.
VI. Conclusion

This paper presents the first study of a PWPF modulated thruster control using the technique of input shaping. The control object is the FSS at the U.S. Naval Postgraduate School. An analytical model of the FSS is developed to identify system frequencies. A detailed analysis of the PWPF modulator is performed to study the impact of modulator parameters on its performance. The PWPF modulator analyses reveal a narrow but effective tuning range for some modulator parameters. Subsequent investigations using a two-stage input gain validate the effectiveness of this technique. Use of the recommended design parameter ranges avoids excessive phase lag, minimizes thruster cycles, and keeps propellant use to a minimum. A command input shaper is designed and integrated with the PWPF modulator. Robustness analyses are performed to show the insensitivity of PWPF modulated input shapers to frequency and damping uncertainty. Numerical simulations performed on an eight-mode model of the FSS demonstrate the efficacy of the variable-amplitude shaped command with PWPF modulation.

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References