DAMPING SYNTHESIS FOR FLEXIBLE SPACE STRUCTURES
USING COMBINED EXPERIMENTAL AND ANALYTICAL MODELS

M.L. Soni*
University of Dayton Research Institute
Dayton, Ohio

B.N. Agrawal**
International Telecommunications Satellite Organization
Washington, D.C.

Abstract

A procedure is presented for modal and damping synthesis of flexible space structures from subsystem tests and/or analyses. In view of the various damping mechanisms operative in a space structure, their contribution to total system damping and compatibility with dynamic testing procedures, the spacecraft subsystems are identified as joints and components. The dynamics of joint subsystems is represented via its stiffness, inertia, and damping properties in physical displacement coordinates. Simple static influence coefficient and cyclic hysteresis tests are used for the determination of joint dynamic properties from experimental measurements. Component dynamics is represented in terms of generalized coordinates, defined to include natural vibration modes, with any arbitrary interface boundary conditions. Ritz vectors, physical displacements, or any other set of linearly independent admissible vectors that is convenient for the experimental and/or analytical characterization of the component. An existing damping synthesis method based on empirical correlation of subsystem stored and dissipated energies is modified to permit the use of measured resonance damping data, thus allowing any arbitrary variability of modal damping data. The results of the developed modal and damping synthesis procedures are verified by using a representative flexible space structure including structural joints.

Introduction

Structural damping plays an important role in the stability and control of flexible space structures. Owing to their size and the nature of damping mechanisms involved, the prediction of damping for such structures is necessarily a combined analytical and experimental effort. A number of component dynamic synthesis methods have been developed in the past, however, a very few studies have been reported concerning the success of these methods.

In a literature review of the subject reported in Reference 1, it was noted that the structural joints are the principal energy dissipators in a space structure, in fact, their contribution to system damping exceed that due to other subsystems by at least an order of magnitude. However, the dynamic characterization of the joint poses insurmountable difficulties when one attempts to use the modal survey procedures. The complex stiffness and dissipation characteristics combined with its size and relative rigidity in the frequency range of interest preclude their modal testing. Beam, plate, or shell-like subsystems of a space structure on the other hand pose no particular problems, and are amenable to conventional modal testing procedures.

The methods of damping synthesis developed in References 2 and 3 do not provide adequate damping synthesis owing to the fact that the models of subsystems damping required by them are often difficult to obtain from experiments. Thus, for example, coupled modal damping matrices cannot be obtained for practical joints. Also the subsystem display a great amount of variability of damping from mode to mode making it impossible to interpolate damping values for off-resonance conditions. The two methods of damping synthesis and their shortcomings are also reviewed in Reference 1.

The present paper present an improvement over the existing methods of damping synthesis in the following respects. First, it combines the best features of both methods of damping synthesis, the matrix method [2] and the energy method [3], in as much as a space structure contains subsystems which require more than one type of damping characterization. Next, the energy method of damping synthesis is modified so as to be able to uses the measured damping data directly, without having to project off-resonance damping.
information from measured resonance damping data, this allows any arbitrary variation of subsystem modal damping from mode to mode. Spacecraft joints are represented in terms of their physical coordinates. Since a joint is relatively rigid in the system frequency range, its modal testing is not required. Its stiffness characteristics are derived from influence coefficient tests and its energy dissipating characteristics are derived from load-deflection hysteresis curves obtained under forced cyclic loading conditions. These tests are conducted at excitation and frequency levels a joint is likely to be subjected to when vibrating as a part of the system.

In the following sections the formulation of the synthesis procedure and joint modeling and its experimental characterization are described. The results of a verification study are also presented.

**Damping Synthesis Formulation**

In the present work we assume that damping does not alter subsystem modes and frequencies and use these undamped properties to obtain built-up system modes, frequencies, and stored energy distributions. The case involving heavily damped subsystems will be addressed in a future work. A structure is partitioned into subsystems identified as components and joints. All subsystems may be considered as either joints, or components connected through fictitious joints: the restriction being that no two joints or components may be connected directly. Space structure subsystems quite naturally fit into the above identification scheme. The components are characterized in terms of their dynamic properties in generalized coordinates such as natural vibration modes, static deflection functions, Ritz vectors, etc., and associated inertia, stiffness, and damping properties. No restrictions are place on the orthogonality of the generalized coordinates, as long as they are from a complete set and are linearly independent. With these conventions, the system synthesis transformation and the coupled system dynamics can be obtained as follows.

The equations of motion of an uncoupled subsystem may be written in the physical coordinates as

\[
[M(s)] \ddot{X}(s) + [C(s)] \dot{X}(s) + [K(s)] X(s) + [H(s)] \dot{X}(s) = F(s)(s)
\]

where \(M, C, K, \) and \(H\) are, respectively, the inertia, viscous damping, stiffness, and hysteric (structural) damping matrices. \(X(s)\) is the displacement vector, \(F(s)\) is the vector of forces including interactive forces due to adjacent systems, and \(s\) is the Laplace parameter. In the following, the superscripts \(s = a\) is used to identify the component and \(s = j\) is used to identify the \(j\)th joint. Equation (1) includes viscous and hysteretic damping. For arbitrary mechanisms and distributions, the damping is specified through an energy correlation of damping data in the manner described in Reference 3.

Equation (1) as it stands represents the dynamic properties of a joint substructure. For a component, we define a generalized modal matrix \(\phi(a)\) extending the definition of

\[
-\dot{x} = \phi(a) \xi
\]

classical modal matrix of the eigenvectors of Equation (1), the column vectors of \(\phi(a)\) are generalized displacement functions such that the vectors are linearly independent, and a linear combination of the vector is capable of representing the elastic motion of the component. The matrix \(\phi(a)\) can thus include normal modes, static deflection functions polynomials, etc. The physical displacement of the component is then

\[
X(a) = \phi(a) \xi(a)
\]

(2)

by definition. In the following, \(\phi(a)\) and \(\xi(a)\) are referred to as

\[
-\dot{x} = \phi \xi
\]

simply the modal matrix and modal coordinate vectors, respectively. Equation (1), together with Equation (2), leads to the equation of motion of the component is generalized coordinate \(\xi(a)\) as

\[
M(a) \ddot{\xi}(a) + C(a) \dot{\xi}(a) + (K(a) + iH(a)) \xi(a) = F(a)(a)
\]

(3)

where \(M(a) = \phi^T \phi\), \(C(a) = \phi^T C \phi\), \(K(a) = \phi^T K \phi\), \(iH(a) = \phi^T H \phi\), etc., are the dynamic property matrices in generalized coordinates, and
depending upon the type of generalized displacement functions chose, the matrices may be diagonal, banded, or fully populated.

The coupled system equations of motion are obtained by considering the continuity of displacement field \( \tilde{\mathbf{x}}(s) \) across the substructure interfaces. Thus, the displacements of a point when considered on adjacent substructures are

\[
\tilde{\mathbf{x}}(j)|_C = \tilde{\mathbf{x}}(a)|_C
\]

where the joint and component connection interface is intended, and \( \tilde{\mathbf{x}}(j)|_C \) and \( \tilde{\mathbf{x}}(a)|_C \) are, respectively, the partitions of the joint and component displacement vectors corresponding to the interface degrees of freedom. Using Equation (2) in Equation (4) leads to a constraint relation between the joint connection point displacements and the generalized modal coordinates of the component as

\[
\tilde{\mathbf{x}}(j)|_C = G(a)|_x
\]

The constraint relation given by Equation (5) results into an exact component synthesis transformation. The extended definition of component modal coordinates, Equation (2), makes the synthesis procedure applicable to arbitrary component interface boundary conditions. The extended definition of the synthesis procedure leading to system modes and frequencies is straightforward: the details may be found in Reference 4.

Synthesis of substructure viscous and hysteretic damping matrices is achieved through system coordinate coupling transformation \( \mathbf{T} \) and system modal matrices \( \psi \). Thus

\[
\mathbf{H}_\eta = \psi^T \mathbf{T} \mathbf{C}_\eta \mathbf{T}^T \psi
\]

and are the system viscous and hysteretic damping matrices in coupled system modal coordinate \( \eta \). The matrix \( \mathbf{H}_\eta \) is intended at a given system frequency only.

Synthesis of the empirical damping laws is affected as follows (Reference 3). The dissipation of energy function is obtained from experiment and curve fitting techniques as

\[
D(s) = a(s)^2 \beta
\]

where \( D(s) \) is the energy dissipation per cycle, \( T(s) \) is the maximum stored energy, and \( a \) and \( \beta \) are the coefficients of the damping law. Following system mode and frequency synthesis, the substructure stored energy is recovered as

\[
\tilde{T}(s) = \tilde{\mathbf{x}}(s)^T \mathbf{H}_\eta \tilde{\mathbf{x}}(s) / 2a(s)^2 \]

where \( a(s) \) is the peak amplitude of vibration of the substructure. The corresponding dissipated energy is obtained from Equation (7). The system damping ratio is then given as

\[
\xi_n = D_n^* / 4\pi T_n
\]

where

\[
D_n^* = \sum_{s=1}^{N} D(s) \left( a(s)/a_0 \right)^2
\]

\[
T_n = \sum_{s=1}^{N} \tilde{T}(s) \left( a(s)/a_0 \right)^2
\]

\( a_0 \) is the peak amplitude of the system mode vector and \( N \) is the total number of subsystems. When damping correlations of the type given by Equation (7) are available for each vibration mode of the components and deformation modes of the joints, the system dissipated energy may be calculated more accurately as

\[
D_n^* = \sum_{j=1}^{NJ} \sum_{k=1}^{MC} \sum_{l=1}^{NJ} \sum_{k=1}^{MC} a_j^{(I)} \left( \lambda_j (J) \right) b_j^{(K)}
\]

where \( a_j^{(I)} \) and \( b_j^{(K)} \) are the experimentally determined parameters of the damping law for the \( I \)th deformation mode of the \( J \)th subsystem. \( NJ \) and \( NC \) are respectively the number of joints and number of components in the system and \( MJ \) and \( MC \) are the number of deformation modes and vibration modes of
each joint and component, respectively. SE represents stored energy.

The Equation (10) represents an improvement over the energy method of Reference 3 in that it uses the measured damping data directly. The method of Reference 3 required grouping of modes and reduction of measured damping data to system frequencies. In Equation (10), the joint damping parameters \( a \) and \( b \) can be obtained in each of its six fundamental deformation modes (axial, transverse and inplane shear, torsion, and transverse and inplane flexure) at excitation and frequency levels as seen by the joint when vibrating as a part of the total system. The component damping parameters are obtainable from resonance measurements.

### Joint Modeling Procedure

Deployable structural joints, owing to their design and complex energy dissipation mechanisms, present a formidable analytical task. Due to these reasons a phenomenological modeling approach is adopted in the present work. The joint model used is basically a three-dimensional version of a two parameters Kelvin-Voigt solid with viscous or hysteretic damping. With a proper choice of the spring and damping parameters the model may be used in an approximate way to represent the overall axial, flexural, torsional, and shear behavior of a physical joint with multiple degrees of freedom. The representation of the joint in terms of a two-connection point model with three translational and three rotational flexibilities is made in view of the relatively small size of the joint and the connection interface in comparison to the major dimensions and flexibilities of the components to which it is connected. The assumption is also necessary to facilitate its experimental characterization.

The joints, as any other subsystem of a built-up structure, undergo a forced vibratory motion when vibrating as a part of the built-up system. The joint natural frequencies are, however, several orders of magnitude higher than the system frequencies of concern. In view of this and considering the fact that joint damping depends upon frequency and amplitude of vibration, the joint stiffness and damping parameters required are obtained from nonresonance forced vibration tests of the joints. Ideally these tests should be conducted at the system frequency and amplitudes of vibrations. In the present work an iterative approach was adopted since the system properties are not known a priori. Joint stiffness coefficients are measured at arbitrarily low frequency cyclic loading conditions. These properties are then used to obtain system modal properties. Joints tests are repeated at these improved frequencies and amplitudes of vibrations. Joint damping value is obtained from the measurement of cyclic energy dissipation and the peak energy stored. These tests are repeated for each system frequency and for each of the six fundamental deformation modes of the joints.

In the following, an example problem is described in which the above procedure is applied to characterize joints and subsequently obtain system damping synthesis.

### Example Problem

A representative flexible spacecraft appendage incorporating realistic deployable joints is used to verify the accuracy of the joint modeling and system damping synthesis procedures developed in the proceeding sections. Figures 1a and 1b show the assembled structure and close-up view of a typical joint, respectively.

![Representative Flexible Spacecraft Appendage - Built-Up System](image-url)
Figure 1b

Representative Flexible Spacecraft Appendage - Structural Joint

The joint shown in Figure 1b is a simple two link hinge designed to simulate the hinge mechanism used in INTELSAT V solar array structure. It consists of a spring-loaded pin, which slides into a cavity to lock the hinge in deployed position. The hinge is movable when the lock-pin is withdrawn from the cavity. Due to dimensional tolerances there is a free play at the two hinge pins. This free play was reduced by employing set screws at the hinge pins. Also the joints were preloaded in order to enable cyclic loading without introducing additional free play at the joint and loading fixture interface. An MTS axial load frame and computer controller programmed in load control was used for testing in every deformation mode of the joints. Extensometers were applied to measure deformations. The loading was time harmonic. When steady state response was reached for a given test, the load deformation cycle was randomly captured by the MTS computer system and routed to an X-Y plotter to record the results. Figure 2 shows some typical hysteresis loops of a joint. In most part the hysteresis loops resemble those of a viscoelastic structure. As expected, the damping values for the joint are considerably higher than those of a viscoelastic structure without any frictional interfaces. Table 1 gives the joint damping (loss factors) and stiffness coefficients for the three predominant deformation modes. The inplane and transverse deformation modes, respectively, refer to the planar and transversal directions of the built-up assembly. Details of joint tests may be found in Reference 5.

Table 2 lists the modal frequencies and loss factors for first several modes of the yoke and the panels.

Table 1

<table>
<thead>
<tr>
<th>Mode of Deformation</th>
<th>Excitation Frequency (Hz)</th>
<th>Loss Factor</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial</td>
<td>1</td>
<td>.008 - .025</td>
<td>214,700 lb/in</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>.011 - .017</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>.032 - .034</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>.077 - .079</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>.150 - .152</td>
<td></td>
</tr>
<tr>
<td>Transverse Bending</td>
<td>.0005 - .006</td>
<td>33,100 in-lb/ rad</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>.002 - .013</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>.017 - .027</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>.06 - .084</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>.165 - .17</td>
<td></td>
</tr>
<tr>
<td>Inplane Bending</td>
<td>.022 - .04</td>
<td>15,900 in-lb/ rad</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>.005 - .009</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>.013 - .028</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>.06 - .068</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>.121 - .143</td>
<td></td>
</tr>
</tbody>
</table>
Results of system modal and damping synthesis using the formulation of Section 2 are given in Table 3. Results of direct system measurements are also given in the table. Combined modified energy and matrix methods of damping synthesis were used in calculating system damping values. The results of matrix synthesis alone are in serious error since an unique damping matrix for joints valid for all system frequencies of interests cannot be defined. The classical energy method of damping synthesis could not be performed since the required modal damping versus stored energy correlation for joints was unavailable from tests.

Table 3
System Modal Properties

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Built-Up System Modal Test</th>
<th>System Synthesis from Subsystem Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Hz</td>
<td>Loss Factor</td>
<td>Frequency Hz</td>
</tr>
<tr>
<td>First Bending 1.38</td>
<td>.0069</td>
<td>1.34</td>
</tr>
<tr>
<td>Second Bending 8.84</td>
<td>.005</td>
<td>9.42</td>
</tr>
<tr>
<td>First Torsion 9.46</td>
<td>.003</td>
<td>9.72</td>
</tr>
<tr>
<td>Third Bending 27.13</td>
<td>-</td>
<td>26.01</td>
</tr>
<tr>
<td>Second Torsion 31.25</td>
<td>-</td>
<td>31.78</td>
</tr>
<tr>
<td>Fourth Torsion 52.06</td>
<td>-</td>
<td>54.43</td>
</tr>
<tr>
<td>Third Torsion 57.06</td>
<td>-</td>
<td>57.26</td>
</tr>
</tbody>
</table>

It should be noted that as a consequence of Equations (9) and (10), the formulation presented in this paper leads to expressions for system damping as explicit functions of the joint deformation modes and component vibration modes. Thus, for example, in the first system mode the system damping \( \eta_1 \) is given as

\[
\eta_1 = 0.02\eta_T^j + 0.253\eta_B^j + 0.392\eta_T^1 + 0.091\eta_Y^1 + 0.102\eta_P^j + 0.105\eta_T^j + \ldots
\]

where the superscripts refer to the joint (J), yoke (Y), and panel (P). \( \eta \) is the loss factor, subscripts denote the joint mode of deformation such as torsion (T) or bending (B), and subscripted numerals denote component mode numbers. The numerical coefficients are simply the fraction of total stored energy contributed by the associated subsystem. Energy dissipation proportional to stored energy is assumed in the above. The utility of the functional relationship of the type lies in assessing the sensitivity of the system damping to subsystem design changes.

Conclusions

A damping synthesis procedure specifically addressing the problems of joint subsystems is presented. It is seen that the modal testing of joints presents difficult problems currently beyond the state-of-the-art modal testing methods. The damping synthesis procedure developed does not require joint modal data, instead more relevant data obtainable in rather simple tests suffice. A procedure is presented for the characterization of joints from experimental measurement. The damping synthesis method presented in this paper also improves the synthesis of component damping by requiring only resonance test data without restricting the variability of modal damping data from mode to mode. A representative flexible spacecraft structure is used to provide a validation of the developed procedure.

The success of the joint modeling procedure presented in this paper is in part due to the structural modifications made in the joints under study. More often the joints have significant amount of free play, giving rise to nonlinear stiffness and impact damping. Further work is needed to characterize more realistic joint behavior and synthesize nonlinear subsystems.

Acknowledgments

This research was conducted by the University of Dayton Research Institute under a contract from the International Telecommunications Satellite Organization. The experimental results presented in this paper were provided by messrs. M.L. Drake, M.F. Kluesener.
M.P. Bouchard, and D.M. Hopkins. Their contributions are gratefully acknowledged.

References


