A SIMPLER APPROACH TO UPDATE SPACECRAFT LAUNCH LOADS

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Abstract

A simpler approach is presented to update launch loads for a spacecraft whose structural dynamic characteristics have been modified during its design phase. The spacecraft dynamic characteristics influence the interface acceleration by introducing anti-resonances (notches) at the spacecraft cantilever frequencies. The proposed approach consists of shifting the anti-resonance frequencies in the interface acceleration in accordance with the changes in the natural frequencies of the spacecraft. It provides a significant improvement in the accuracy of the calculated spacecraft launch loads in comparison with the base drive technique. Numerical examples have been used to validate the approach.

Introduction

Dynamic launch loads for a spacecraft are determined by performing the coupled launch vehicle/spacecraft dynamic analysis. These loads are functions of not only launch vehicle dynamic characteristics and external forces, but also of the dynamic characteristics of the spacecraft. Hence, coupled analyses are performed on each new spacecraft program. A coupled analysis consists of generation of the structural analytic models for the spacecraft and the launch vehicle, calculation of their modal characteristics, coupling of the modes, calculation of system modal characteristics, calculation of time responses of the system to the specified forces, and the use of the response results to calculate spacecraft internal loads.

The coupled analysis is very costly and time consuming (engineering and turnaround time). Due to these considerations, normally two coupled analyses are performed on a spacecraft program. First coupled analysis is performed in the beginning of the program to determine the launch loads by using a preliminary structural model of the spacecraft and the second coupled analysis is normally performed just before launching the first spacecraft to satisfy all spacecraft/launch vehicle structural design requirements. During the design phase, the structural design modifications, although normally small, take place, resulting in the change of the spacecraft dynamic characteristics from those of the preliminary structural model. The launch loads are, however, not normally updated due to cost and time considerations. This results in uncertainty in the adequacy of the structural design. Hence, there is need for a simpler method to update the launch loads without performing an additional coupled dynamic analysis.

During the last decade, several new coupled analysis methods have been developed which have received wide acceptance. However, currently there is no satisfactory "short cut" method. They suffer from complexity, inaccuracy and lack of validation of the approach. One approach, known as the base drive technique, is to use the interface accelerations calculated from old coupled analysis for the new spacecraft design. This approach neglects the modification of the interface acceleration due to changes in the dynamic characteristics of the spacecraft. This method, as will be discussed in the later sections, can result in significant errors in launch load calculations. The other methods are the perturbation techniques and shock spectrum techniques. None of these methods have proved promising because they do not provide significant reduction in analytical effort in comparison with the coupled analysis.
Simpler Approach

This method is based on the postulation that if a coupled analysis has been already performed for a new spacecraft, then it should not be necessary to perform it again if small changes have taken place in the structural dynamic characteristics of the spacecraft during its design phase. It should be possible to modify the spacecraft interface acceleration independently by taking into account these changes in the spacecraft dynamic characteristics. In order to modify the interface acceleration, however, it is necessary to identify the effect of spacecraft dynamic characteristics on it explicitly. This section provides a detailed discussion on these effects. The frequency domain analysis has been used to provide insight into these effects.

The dynamic interaction between the payload and the booster can be expressed in terms of reaction forces from the payload at the booster interface. Therefore the booster is subjected to two types of forces: external forces and reaction forces from the payload. The booster/payload interface acceleration can be written as a summation of accelerations from these forces as:

\[ \ddot{x}_B(t) = \ddot{x}_{BF}(t) + \ddot{x}_{BR}(t) \]  

(1)

where

\[ \ddot{x}_{BF}(t) = \text{booster/payload interface acceleration due to external forces, neglecting the reaction forces from the payload.} \]

\[ \ddot{x}_{BR}(t) = \text{booster/payload interface acceleration due to the reaction forces from the payload.} \]

Taking the Fourier transform of Eq. (1) yields,

\[ \hat{X}_B(\omega) = \hat{X}_{BF}(\omega) + \hat{X}_{BR}(\omega) \]

(2)

where

\[ \hat{X}_B(\omega) = \text{acceleration of booster/payload system in the absence of payloads.} \]

\[ \hat{X}_{BF}(\omega) = \text{acceleration of booster/payload system due to external forces, neglecting the reaction forces from the payload.} \]

\[ \hat{X}_{BR}(\omega) = \text{acceleration of booster/payload system due to the reaction forces from the payload.} \]

The acceleration \( \hat{X}_{BF}(\omega) \), which represents the launch vehicle response in the absence of payloads, need to be calculated only once. The transfer function \( \hat{H}(\omega) \) is a function of payload dynamic characteristics and represents the modification of the interface acceleration due to payload reaction forces. Therefore, to update launch loads for a spacecraft, we need to modify \( \hat{H}(\omega) \) as a function of its new dynamic characteristics.

Transfer Function

A simple explanation of the characteristics of the transfer function \( \hat{H}(\omega) \) can be given in terms of the function of a Dynamic Vibration Absorber\(^5\), invented by Frahm in 1909. It consists of a spring-mass system and is attached to the body whose vibrations are to be reduced. Its natural frequency is tuned to the frequency of the disturbing force. For the special case of no damping in the damper, the body will not vibrate at all. The vibration absorber will vibrate in such a way that its force on the body will be equal and opposite to the disturbing force, resulting in the cancellation of the disturbing force. In the presence of damping, the body will vibrate but will have significant attenuation at the natural frequency of the damper. Applying this analogy to a booster/payload system, the booster as the body and the payload as the vibration damper, the interface acceleration will be attenuated at the payload natural frequencies. This phenomenon is known as "notching" in sinusoidal testing and is the basis of notching the vibration specification at the payload primary natural frequencies.

The influence of the payload dynamic characteristics on the interface acceleration and transfer function can be further studied by using the following analysis which was developed by Payne\(^6\) and is known as the Impedance Technique. The equations of motion of a uncoupled booster and payload system, as shown in Figure 1, can be written as
Substituting Eq. (5) into Eq. (4), solving for interface acceleration, and taking its Fourier transform, we get

\[
\{\ddot{X}_B^B(\omega)\}^F = \{\hat{A}(\omega)\}\{F\}
\]

where

\[
A(\omega) = \begin{bmatrix} \Phi_I^B \end{bmatrix} \begin{bmatrix} -\omega^2/(\omega_B^2 - \omega^2) + i2\xi_B\omega_B \omega \end{bmatrix} \begin{bmatrix} \Phi_I^F \end{bmatrix}
\]

where \(\omega_B\) are the natural frequencies of the free-free modes of the booster. Matrix \(A(\omega)\) is the transfer function between the external force and the interface acceleration, similarly,

\[
\begin{bmatrix} R_B^B(\omega) \\ R_P^B(\omega) \end{bmatrix} = \begin{bmatrix} B(\omega) \end{bmatrix} \begin{bmatrix} \Phi_I^B \\ \Phi_I^F \end{bmatrix}
\]

where

\[
B(\omega) = \begin{bmatrix} \Phi_I^F \end{bmatrix} \begin{bmatrix} -\omega^2/(\omega_B^2 - \omega^2) + i2\xi_B\omega_B \omega \end{bmatrix} \begin{bmatrix} \Phi_I^B \end{bmatrix}
\]

Matrix \(B(\omega)\) is the transfer function between the reaction forces at the interface and the interface acceleration.

Next, we have to determine reaction forces \(R_P\) from payload equations. The equations of motion of the payload are

\[
\begin{bmatrix} M_P^N \\ M_P^I \end{bmatrix} \{x_P^P\} + \begin{bmatrix} C_P^N \\ C_P^I \end{bmatrix} \{v_P^P\} + \begin{bmatrix} K_P^N \\ K_P^I \end{bmatrix} \{x_P^P\} = \begin{bmatrix} 0 \\ -F_P \end{bmatrix}
\]

where the \(F\)-vector represents the response of the booster due to the external force \(F_B\) and \(R\)-vector represents the response due to the reaction force \(R_B\) at the interface. The subscript \(N\) represents the non-interface degrees of free. The matrix \([\Phi]\) consisting of model vectors for free-free modes of the booster and \(q_B\) are the corresponding model coordinates.
The payload motion in terms of model coordinates can be expressed as

\[
\begin{pmatrix}
    \dot{X}_N^P \\
    X_I^P
\end{pmatrix} = \left[ \Phi_N^P \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right] + \left[ \Phi_N^P \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right]
\]

(11)

where \([\Phi_N^P]\) is the model vector matrix for the cantilever modes of the payload and \([q_N^P]\) are corresponding modal coordinates.

By substituting Eq. (11) into Eq. (10), the resulting equations are

\[
\begin{pmatrix}
    \dot{q}_N^P \\
    \dot{M}_1^P \\
    \dot{M}_2^P
\end{pmatrix} + \left[ \Phi_N^P \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right] + \left[ \Phi_N^P \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right] = - \left[ \Phi_N^P \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right]
\]

(12)

Assuming the interface to be statically determinate, the reaction forces \(R_P\) from the above equations are

\[
\{ R_P(\omega) \} = \left[ C(\omega) \right] \{ \dot{X}_I^P(\omega) \}
\]

(13)

where

\[
\left[ C(\omega) \right] = \left[ M_2^P \right]^T \left[ \Phi_N^P \right] \left[ \frac{-\omega^2}{\omega^2 - \omega_1^2} + \frac{i2\zeta_P\omega_1}{\omega_1^2 - \omega^2} \right] \left[ S_P \right] \left[ \dot{X}_I^P \right] \left[ q_N^P \right]
\]

(14)

and \(\omega_1\) are the cantilever natural frequencies of the payload.

Substituting Eq. (13) into Eq. (8), we get

\[
\begin{pmatrix}
    \dot{X}_I^P(\omega) \n
\end{pmatrix} = \left[ B(\omega) \right] \left[ C(\omega) \right] \{ \dot{X}_I^P(\omega) \}
\]

(15)

\[
\begin{pmatrix}
    \dot{X}_I^P(\omega) \n
\end{pmatrix} = \left[ B(\omega) \right] \left[ C(\omega) \right] \{ \dot{X}_I^P(\omega) \}
\]

(15)

\[
\begin{pmatrix}
    \dot{X}_I^P(\omega) \n
\end{pmatrix} = \left[ B(\omega) \right] \left[ C(\omega) \right] \{ \dot{X}_I^P(\omega) \}
\]

(15)

where

\[
[H(\omega)] = \left[ I \right] - \left[ B(\omega) \right] \left[ C(\omega) \right]^{-1}
\]

(16)

substituting Eq. (6) into Eq. (15), we get

\[
\begin{pmatrix}
    \dot{X}_I^P(\omega) \n
\end{pmatrix} = \left[ B(\omega) \right] \left[ C(\omega) \right] \{ \dot{X}_I^P(\omega) \}
\]

(17)

The elements of transfer functions \(H(\omega)\) and \(\tilde{H}(\omega)\) can be expanded in the following form

\[
\tilde{H}(\omega) = \tilde{K}_{ij} \frac{N}{\pi} \left[ \frac{(\omega_1 - \omega_1^2) + i2\zeta_P\omega_1}{\omega_1^2 - \omega^2} \right]
\]

(18)

\[
\tilde{H}(\omega) = \tilde{K}_{ij} \frac{N}{\pi} \left[ \frac{(\omega_1 - \omega_1^2) + i2\zeta_P\omega_1}{\omega_1^2 - \omega^2} \right]
\]

(19)
where $H_{ij}$ and $H_{ji}$ are the elements in the $i$th row and $j$th column of the transfer functions $H(\omega)$ and $\tilde{H}(\omega)$, respectively; $N_N$, $N_P$ and $N_S$ are the numbers of degrees of freedom of the booster, payload, and the coupled booster/payload system; and $\omega_{nj}$, $\omega_{pi}$ and $\omega_{si}$ are the natural frequencies of the $i$th mode of the booster, the payload and the coupled booster/payload system.

From the Eq. (18), it can be noted that the elements of the transfer function $H(\omega)$ have anti-resonance (zeros) at the natural frequencies of the booster and the payload, and resonances (poles) at the natural frequencies of the coupled booster/payload system. Similarly from Eq. (19), the elements of $H(\omega)$ will have anti-resonance (zero) at the natural frequencies of the payload and resonance (poles) at the natural frequencies of the coupled booster/payload system. Therefore, if the natural frequencies of the payload change, the anti-resonances in the interface acceleration corresponding to the payload natural frequencies will also shift accordingly. The resonance frequencies in the interface acceleration will also shift due to changes in the natural frequencies of the coupled booster/payload system.

To calculate the interface acceleration for the new payload, one approach as suggested by Payne6, would be to recalculate the new transfer matrix. This approach will not, however, result in significant reduction in the analytical effort in comparison with the new coupled analysis. The analytical effort can be, however, reduced significantly if we limit the modification of the transfer function in the frequency ranges which are critical to the payload response. The transfer function between the payload response and the interface acceleration peaks at the cantilever frequencies of the payload. Therefore, it is necessary to modify the interface acceleration at these frequencies to take into account the changes in the spacecraft dynamic characteristics. Otherwise, a significant error will be introduced in the calculated payload response. The interface acceleration has peaks at the natural frequencies of the coupled booster/payload. However, the transfer function between the payload response and the interface acceleration does not have peaks at these frequencies. Therefore, it is not necessary to shift the resonances (peaks) of the interface acceleration in accordance with the change in the natural frequencies of the coupled system. However, it is critical that these interface acceleration peaks do not coincide with the natural frequencies of the modified payload. Otherwise, the payload response will be unrealistically high. There is a risk of occurrence of this condition in the use of the base drive technique i.e. use of interface acceleration from the coupled analysis of the other spacecraft.

The proposed approach consists of modifying the interface acceleration at the payload natural frequencies, shifting the anti-resonance frequencies, in accordance with the updating of payload natural frequencies. This approach will result in significant improvement in the accuracy of the calculated payload response in comparison with the basic drive technique.

In comparison to performing a new coupled analysis, the analytical effort will be significantly reduced because it does not require the calculation of the new coupled booster/payload natural frequencies and the transfer function matrix. It should be noted, however, that the accuracy of this method depends on the degree of changes in the spacecraft dynamic characteristics. Therefore, this approach is mainly useful to update the launch loads where only minor change have taken place in the dynamic characteristics of the spacecraft during its design process. The validity of this approach is further demonstrated by the numerical example in the next section.

**Numerical Examples**

The proposed approach is demonstrated by performing dynamic analyses on a simple booster/payload model. The objectives of the analyses are: (a) to analyze the influence of the natural frequencies of a payload on its interface acceleration and its transfer function $H(\omega)$, and (b) to show that the errors in the prediction of the response of a payload can be significant if the interface acceleration from other payload coupled analysis were used.

**Models**

The payload model for the analysis is shown in Figure 2. It consists of two masses, two springs and the base is assumed to be fixed. The analysis has been performed for two sets of payload parameters. The parameters for phase models are as follows:

```plaintext
# Parameters
mass1 = 1000 kg
mass2 = 500 kg
spring1 = 100 N/m
spring2 = 200 N/m
base = 1000 N

# Analysis
Analysis1 = perform_analysis(mass1, mass2, spring1, spring2, base)
Analysis2 = perform_analysis(mass1, mass2, spring1, spring2, base)
```
Figure 2 - Payload Model

Model I

\[ \begin{align*}
M_1 &= 1.753 \times 10^3 \text{ kg (10 lb-s}^2/\text{in)} \quad C_1 = 0.01 \\
M_2 &= 8.765 \times 10^3 \text{ kg (50 lb-s}^2/\text{in)} \quad C_2 = 0.01 \\
K_1 &= 3.506 \times 10^6 \text{ N/m (2 x 10}^4 \text{ lb/in)} \\
K_2 &= 1.227 \times 10^7 \text{ N/m (7 x 10}^4 \text{ lb/in)} 
\end{align*} \]

The cantilever natural frequencies are

\[ f_1 = 5.033 \text{ Hz} \quad f_2 = 8.422 \text{ Hz} \]

Model II

\[ \begin{align*}
M_3 &= 7.012 \times 10^4 \text{ kg (400 lb-s}^2/\text{in)} \\
M_4 &= 8.785 \times 10^4 \text{ kg (50 lb-s}^2/\text{in)} \\
K_3 &= 2.4542 \times 10^7 \text{ N/m (140,000 lb/in)} \\
K_4 &= 1.283639 \times 10^6 \sin \omega t \quad (\omega = 0 - 50 \text{ Hz}) 
\end{align*} \]

The free-free mode natural frequencies are

\[ f_1 = 0.01 \quad f_2 = 4.211 \text{ Hz} \]

Analyses and Discussions

The dynamic analysis has been performed for the following cases.

Case I - Booster Response

The dynamic response of the booster, in the absence of the payload, is calculated due to external force \( F \). The magnitude of \( X_B(\omega) \), Fourier transform of the interface acceleration, is shown in Figure 4. Using this interface acceleration, the response of the payload model I is calculated. The magnitude of \( P \), \( X_B(\omega) \), response of the mass 1, is shown in Figure 5.

Figure 4 - Interface Acceleration in The Absence of Payload

The booster model is shown in Figure 3. It consists of two masses with a spring in between. It is subjected to sinusoidal force at one end. The other end provides interface to the payload base. The parameters of the model are as follows:
Case II - Coupled Booster/Payload Model I Analysis

The dynamic response of the coupled booster/payload mode I is calculated due to the external force. The free-free natural frequencies of the coupled system are:

- $f_1 = 0$, $f_2 = 3.92$ Hz, $f_3 = 5.56$ Hz, and $f_4 = 8.5$ Hz.

The magnitudes of $X_1(\omega)$, the interface acceleration; $X_1(\omega)$, response of mass 1 of the payload, and $H(\omega)$, the transfer function, are given in Figures 6, 7, and 8, respectively. These results clearly show that the interface acceleration and the transfer function have notches (anti-resonances) at the payload cantilever frequencies (5.0 Hz and 8.42 Hz) and peaks (resonances) at the natural frequencies of the coupled system.

Case III - Coupled Booster/Payload Model II Analysis

The dynamic analysis under this case is similar to the analysis in case II, except that the parameters of the payload model II are used instead of model I. The difference in the payload parameters is only in the stiffness $K_2$ which is lower in the model II. The free-free natural frequencies of the coupled system are:

- $f_1 = 0$, $f_2 = 3.88$ Hz, $f_3 = 5.30$ Hz, and $f_4 = 8.35$ Hz.

The magnitudes of $X_1(\omega)$ and $H(\omega)$ are given in Figures 9 and 10, respectively. The comparison of the interface accelerations and the
transfer functions for cases II and III indicate that the shift in the frequencies of the anti-resonance (notches) and resonances (peaks) correspond to the changes in the cantilever natural frequencies of the payload and free-free mode frequencies of the booster/payload, respectively. Therefore, if the changes in natural frequencies of a payload are known, the interface acceleration can be modified by shifting the anti-resonances without performing a new coupled analysis.

Case IV: Dynamic Response of the Payload Model I by Using Interface Acceleration from Case II

To study the errors introduced in the calculation of launch loads by using interface acceleration from other payload coupled analyses, the dynamic response of the payload model I is calculated by using interface acceleration from the coupled analysis of payload II, i.e., from case III. The approximate response of the mass $l_1 X_i^E$, calculated under this case is superimposed on the exact response from case II in Figure 11. These results indicate that the peak acceleration of mass 1 from the approximate solution, case IV, is greater than that from the exact solution, case II, by a factor 2. Table 1 gives the comparison of the magnitudes of interface accelerations at payload 1 natural frequencies from approximate solutions, case I and IV, with those from the exact solution, case II. These results clearly indicate that the exact interface accelerations at payload frequencies are significantly lower than those from the approximate analyses. Therefore the use of interface acceleration from the coupled analysis of another payload can result in significant errors in the prediction of launch loads for a payload.
Summary and Conclusions

This paper presents a simpler approach to update launch loads for a payload whose structural dynamic characteristics have been modified during its design phase. The payload dynamic characteristics influence the interface acceleration by introducing anti-resonances (notches) at the payload cantilever natural frequencies. The transfer function between the payload response and the interface acceleration peaks at these frequencies. Therefore, it is necessary to have accurate values of the interface acceleration at these frequencies. Since the interface acceleration at these frequencies has anti-resonances, they should be shifted if the dynamic characteristic (natural frequencies) of the payload change. The interface acceleration also has resonances at the natural frequencies of coupled booster/payload system which will also change with the modification of payload dynamic characteristics. It is, however, not critical to shift these resonance frequencies of the interface acceleration because the transfer function between the payload response and the interface acceleration does not have peaks at these frequencies. Based on these reasonings, the proposed approach consists of shifting the anti-resonance frequencies in the interface acceleration in accordance with the change in the natural frequencies of the payload. The base drive technique, which does not take into account the influence of payload dynamic characteristic on the interface acceleration, can introduce significant errors in the calculated payload response.

The proposed approach provides a significant improvement in the accuracy of the calculated payload response in comparison with the basic drive technique. The analytical effort is also significantly reduced in comparison with the coupled analysis because the present approach does not require the calculation of new coupled booster/payload natural frequencies and new transfer function matrix. The numerical example have validated the basis of this approach. The numerical results show that the use of approximate interface acceleration can result in significantly higher payload response. It should be noted that in order to demonstrate explicitly the effects of payload dynamic characteristics on the interface acceleration, simple structural models have been used in the numerical examples. Further investigation is, however, required to demonstrate this approach on a complex structural models of launch vehicles and spacecraft with a complex forcing function.

References


