The Kaplan and Garrick Definition of Risk and its Application to Managerial Decision Problems

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Abstract

This paper discusses the application of the Kaplan and Garrick (K&G) definition of risk in the context of the managerial decision problem. These problems have a formulation that provides a different information structure than that originally considered by K&G. A fourth question must to be asked. The decision maker's preferences must be ascertained. *Without knowledge of decision maker preference there is no risk.* The answer to the fourth question gives the decision maker's payoff function, $v(Y)$. A new triple $\{Y, P_Y, v(Y)\}$ replaces the original triple of K&G, $\{F, P_F, Y(F)\}$ and permits *an explicit quantification of risk.*

1 Introduction

Risk has many definitions but one stands out above all the rest by proposing a concise quantitative meaning. It is the definition that forms the basis for many analyses and is the genesis of the approach we have taken in DRMI. This is the definition of risk proposed by S. Kaplan and B.J. Garrick (1981). DRMI uses the three questions of Kaplan and Garrick (K&G), but the context of our problems requires an extension in the form of a fourth question. The subtle, but important differences, in problem context between the managerial decision problem and the engineering safety problem (the focus of the original work of K&G) produce a different information structure. It is crucial that this difference be understood because a doctrinaire application of the K&G definition diverts attention from consideration of important problem elements and obscures the other information. The following presentation attempts to clarify these points. First, we review the basic elements of a decision problem, with attention focused on risk management decisions. Second, we review the K&G problem context to understand the rationale for their three question definition. Third, we address the managerial decision problem context in detail, with special attention to the differences between contexts. Finally, we provide a simple example to illustrate the important points.

2 The Elements of a Decision (Especially Risk Management)

Many managerial decisions are subject to significant uncertainty and risk becomes an overriding factor. A prime example is the allocation of resources in defense organizations. Threats to national security change, sometimes in unimaginable ways. Technological obsolescence occurs rapidly. Weapon systems costs and development times cannot be predicted with certainty. Future costs for the various components of operations and maintenance are uncertain. These circumstances, among many more, lead decision makers to view resource allocation decisions as risk management decisions.

Risk management decisions, as with all decision problems, require definition of three elements [see the statement by Professor Ron Howard at the end of Chpt. 15, pg. 126 in Savage (2009)]:

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1. **A Set of Alternatives.** If you don't have alternatives there is nothing to choose.

2. **An Information Structure.** If you don't see the connection between what you do and what's going to happen you can't make a decision.

3. **Decision Maker Preferences.** If you don't care what happens, you can do anything and it doesn't matter.

The set of alternatives in risk management initially contains only two elements: (1) NO, the risk is "insufficient" to warrant further action and can be ignored; or (2) YES, there is "sufficiently high" risk and it cannot be ignored. Only after accepting the affirmative alternative will we proceed to the management decision problem: What course of action most reduces risk subject to resource constraints? Here there can be many alternatives, generally classified in three broad groups: (1) prevention/avoidance actions; (2) mitigation actions; and (3) "changing the rules" or negotiation with the environment.

The information structure is established by the problem definition. This includes definition of the decision maker's objective and its quantification using a variable called the *outcome of concern.* This is a measure of gain (when the statement of the objective implies "more is better") or a measure of loss (when the statement of the objective implies "less is better"). Also included is the definition of all variables that affect, or explain, the outcome of concern. These variables include those known to be under the control of the decision maker, as well as those beyond the control of the decision maker. Finally, there is specification of the function relating the outcome of concern to the explanatory variables. Probabilistic models for variables and relations complete the problem definition to account for uncertainty.

Decision maker preferences are complicated when uncertainty is present. Uncertainty creates an environment called "decision making under risk". In this environment decision maker preferences literally exhibit "more twists and turns" than decisions made under certainty [see Chris Starmer (2000)]. The simple linear "more is better" or "less is better" payoff function is inadequate to explain decisions and led to the use of a nonlinear *utility function* [see Bernoulli, (1954)]. Recent research exposing the defects of the utility function has led to its replacement with a *prospect function* [Kahneman and Tversky (1979)]. This describes the preferences of the decision maker using a *payoff function* with: (i) a *reference point* separating losses from gains; (ii) *loss aversion* where losses loom larger than gains; (iii) and *decreasing marginal returns* with respect to both losses and gains. The characteristics of a prospect function become critically important in risk assessment. This is especially so for the reference point. For example, suppose the decision maker has a preference for "less rather than more" of the outcome (e.g.,fatalities, econmic loss, etc.). An outcome probability distribution depicting the very high likelihood of very large values for the outcome may cause this decision maker to conclude the situation is "too risky" and demand that "something be done to lower the risk". On the other hand, another decision maker may observe the same distribution but conclude the situation has "low" risk. Why? The first decision maker may have a much lower reference point than the second decision maker. *Without a knowledge of preferences (i.e., the payoff function) the information generated in risk assessment is meaningless.* This last point is of major importance for, as Sam Savage says [Chpt. 7, pg. 53 in Savage (2009)]: "Risk is in the eye of the beholder." What is risky to one person may not be so to another. The need to explicitly recognize the role played by preferences in risk assessment and management is of paramount importance.

3   The Kaplan and Garrick Quantitative Definition of Risk

Hanford, Chernobyl, Three Mile Island and Fukushima Dai-Ichi are names that forever will be associated with potential catastrophe. As a result, nuclear reactor operations have become the subject of public concern for at least 50 years. Assessing the risks to the public of these large, complex engineering systems

forms a major challenge in analysis. One must confront many problems, but most important is developing a detailed description of how a normally operating system can fail or degrade. Thus, it is natural first to ask: "What can go wrong?" Fault trees and similar mathematical structures are used to define an extensive set of (failure) scenarios. Given this list it is next logical to ask "How likely is each to occur?" This requires an exercise in applied probability theory to obtain the likelihood of occurrence of each of the failure scenarios. Finally, one must attempt to predict the consequences of failure, i.e., the outcomes that result for each of the many ways the system can fail. Thus, the final question to ask is: "If it does go wrong, what is (are) the consequence(s)?" The answers to these questions provide the basis for public debate and the exercise of public judgement. The acceptability, or otherwise, of the risk is ultimately a function of the public's risk perception. This is the context out of which grew the K&G quantitative definition of risk.

Stanley Kaplan and B. John Garrick (1981) define risk as the answers to three questions:

1. What can go wrong?

2. How likely is it to go wrong?

3. If it does go wrong, what are the consequences?

The answer to the first question defines a (failure) scenario, denoted by S_i. The answer to the second question is the probability, p_i of the scenario. The answer to the third question is the end *result* or *outcome,* Y_i, generated by the process (sequence of events) described by the scenario. (Note: Kaplan and Garrick use x_i to denote this and refer to it as the "consequence" – a term we reserve for the decision maker's valuation of the outcome – see below.) Thus, risk is portrayed by Kaplan and Garrick as a set of triples:

$$\{S_i, p_i, Y_i\} \tag{1}$$

where $i = 1, 2, \ldots, N$. is employed to distinguish between the different scenarios. N is the total number of scenarios and should be large enough to form a "complete" set (as explained by Kaplan (1997)).

Each scenario is thought of as comprising a chain of events:

$$S_i = \{F_{i1}, F_{i2}, \ldots, F_{iK_i}\}. \tag{2}$$

This form of the scenario is produced immediately from fault tree analyses and similar methods associated with engineering safety studies (i.e., nuclear reactor safety studies). Each scenario represents a unique concatenation of events. If F_{i1} is the initiating even that occurs with probability $p_i(1)$, and all succeeding events in the chain occur with probabilities $p_i(k|k-1)$, then the likelihood of the scenario comprised of K events is

$$p_i = p_i(1) \cdot p_i(2|1) \cdot \cdots \cdot p_i(K_i|K_i - 1). \tag{3}$$

The likelihood of the scenario is identical to the likelihood of the outcome. Thus, the last two elements of the triple specify a probability distribution over the outcome:

$$P_Y = \{Y_i, p_i\}. \tag{4}$$

This is the heart of Probabilistic Risk Assessment (PRA) as advocated by Kaplan and Garrick.

One major and one minor observation must be made before proceeding.

First, the use of the word "consequence" in the third question is unfortunate. While it may be acceptable prose, it leads to confusion. Decision theory reserves the term "consequence" for reference to the decision maker's preferences (i.e., how the decision maker "feels" or "values" the likely outcomes). We know from Ron Howard that preferences are vital to a well-posed decision problem, yet nowhere do K&G query decision maker preferences. Why? Risk assessment in their context ultimately becomes a matter of *public*

policy. The answer to the question: "Is the risk too great?" cannot be answered without knowledge of the public's preferences over the outcomes. This information can only be ascertained through public forums and political institutions. K&G do not address this process. K&G focus *only on the development of the information to which the public applies their preferences* in order to make a decision. This is explicitly recognized by Garrick in his recent book (2008, pg.5).

Second, the scenario concept is *identical* to the concept of *future condition* that we use at DRMI. Both refer to events that have yet to happen and both represent events that influence the outcome of concern. Thus, we can write $F_i = S_i$.and analogous to (4) we obtain:

$$P_F = \{F_i, p_i\}.$$

4 The Managerial Perspective

The context of the managerial decision problem under uncertainty is best described as follows. The central problem is the choice of a variable X so as to make another variable Y behave in a desired way. Y is the outcome of concern and is defined by a function of two types of explanatory variables:

$$Y = f(X, F). \tag{5}$$

X represents the decision variable and is under the control of the decision maker. F is a random variable representing the factors beyond control of the decision maker. The function $f(X, F)$ *is known*. The only information we have about F is its probability distribution, P_F. We refer to F to as the *future condition*. The decision maker chooses X so that Y takes on values over the planning period that are "best" in the sense of the objectives of the decision maker. The "best" decision is conditional on F and is a function of F (i.e. $X = h(F)$) and (5) becomes

$$Y = f(h(F), F) = g(F). \tag{6}$$

Given F and $f(\cdot, \cdot)$ we can determine, Y, the outcome of concern (the "consequence" in Kaplan and Garrick (1981)). In this representation of the system, $g(\cdot)$ is considered known, but F must be estimated subject to its assumed P_F (popularly referred to as an "educated" guess).

This is a much richer information set compared to that assumed by K&S. Here we have knowledge of (5) and (6). Given F we know Y, the resulting outcome of concern (the "consequence" in the terminology of K&S). We also have acess to decision maker preferences. The decision maker may be an individual (e.g., CEO, CFO or CIO, program manager, etc.), a small group of individuals (e.g., an executive board, council, etc.) or an agent for an organization (e.g., the US DoD, Navy, Army, Air Force, etc.). In each case *preferences exist and are articulated*. They are the foundation of the defintion of Y and of what is meant by "best" (for both X and Y).

All of this means we need to interpret differently the three questions of K&S. More importantly, *a fourth question must be asked*!

4.1 The First Question

The first question posed by K&G concerns F. "What can go wrong?" Quite simply, we can "guess wrong" – F can take a value different from what we have assumed and our decision variable, X turns out to be "wrong" (suboptimal) so the resulting $Y = g(F)$ is "wrong"; i.e., takes on a value that is less desirable than would be the case had we "guessed right" with respect to F. The managerial decision problem formulation specifies a direct connection between F and Y through $X = h(F)$. We know, therefore, the "consequences" for any value of F. It matters little if we choose to describe "what goes wrong" in terms of F or in terms of the corresponding Y. What really matters is that we have correctly identified F and that we have correctly constructed (6).

4.2 The Second Question

The second question posed by K&G concerns the probability distribution over F, denoted by P_F. Knowing this distribution, and the system defined by (6), we can always derive the probability distribution P_Y. Practically speaking, this is seldom done using the calculus (as in textbooks on probability, statistics and random variables). In practice we more often use simulation modelling to obtain P_Y given P_F and (6). Therefore, the answer to the second question is P_Y. P_F and P_Y equivalent in the sense that once we know P_F we can quickly obtain the corresponding P_Y. Similar to the first question, it matters little if we choose to describe "how likely it is to go wrong" in terms of P_F or P_Y.

4.3 The Third Question

The answer to the first two questions provide the information necessary to compute the answer to the third question posed by K&G. Given P_F *and* (6) we have P_Y. This is the important implication of the problem context defined by (6). There is no need to ask the third question posed by K&G. It is a question asked and answered in the process of formulating the decision problem; i.e., in the development of (6). There is, however, need to ask a fourth question.

4.4 The Fourth (really the "new" third) Question

While knowledge of Y and P_Y is absolutely essential, risk cannot be assessed *without knowledge of the decision maker's preferences over the probable Y*. We need to know *how the decision maker values to the probable Y. Without preferences the decision maker doesn't care what happens – there is NO risk.* Therefore we ask a *fourth* question with reference to P_Y :

"How do you feel about it? How do you feel about the likely Y?"

This question asks the decision maker to state preferences with respect to the likely Y. What outcomes do you most prefer to avoid? What outcomes do you most prefer to experience? The answer is the decision maker's payoff function, sometimes called the value function (the former name is less confusing in this note),

$$v(Y). \tag{7}$$

This function is *NOT* a utility function [see the extensive review in Schoemaker (1982)]. It is a *prospect function* as described in Kahneman and Tversky (1979): $v(Y) = v^+(Y) + v^-(Y)$ where

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\begin{aligned}
v^+(Y) &= [Y - Y_R]^\beta \text{ if } Y - Y_R > 0 \\
v^-(Y) &= -\lambda |Y - Y_R|^\gamma \text{ if } Y - Y_R < 0
\end{aligned}
$$

when more Y is preferred to less Y. If preferences are reversed so less Y is preferred to more Y then we have

$$
\begin{aligned}
v^+(Y) &= -\lambda [Y - Y_R]^\gamma \text{ if } Y - Y_R > 0 \\
v^-(Y) &= |Y - Y_R|^\beta \text{ if } Y - Y_R < 0.
\end{aligned}
$$

This function is characterized by: (i) a reference point, Y_R, that distinguishes the Y representing gain from the Y representing loss; (ii) loss aversion, represented by the constant, $\lambda > 0$; and (iii) decreasing marginal returns for both losses and gains, represented by the powers $0 \leq \beta < 1$ and $0 \leq \gamma < 1$. Figure 1 is an example where "more is better" and Figure 2 is an example of the "less is better" case ($Y_R = 3$ in both cases).

4.5 A Quantitative Measure of Risk

We find the K&G definition of risk applied in the context of the managerial decision problem produces a new triple:

$$\{Y, P_Y, v(Y)\} \tag{8}$$

the third element of which, $v(Y)$, provides us the information needed to *evaluate risk quantitatively*. Two quantitative measures immediately come to mind: (i) the expected value of the payoff, $E\{v(Y)\}$; and (ii) the probability of loss in terms of the payoff, $\Pr\{v(Y) < 0\}$.
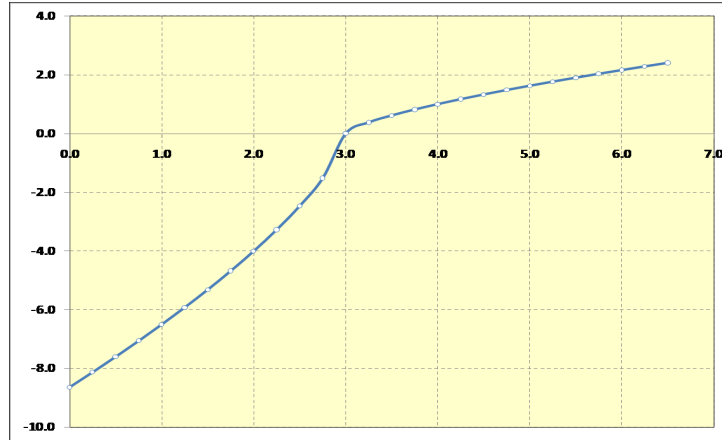

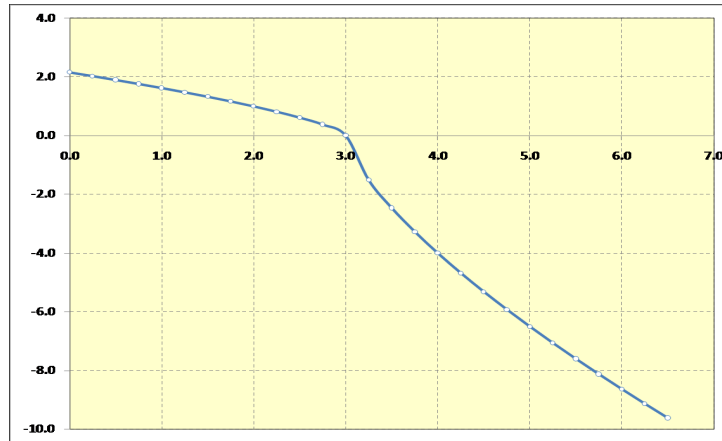Figure 1: Prospect Function $v(Y)$ when more Y is preferred to less Y.

Figure 2: Prospect Function $v(Y)$ when less Y is preferred to more.

4.5.1 Expected Payoff

The most widely used of any measure (see Kahneman and Tversky (1979), Schoemaker (1982), Wakker (2003) and the literature cited in Starmer (2000)), is the expected value of the payoff function[1]:

$$R_1 = E\{v(Y)\} = \sum_{k=1}^{k=K} v(Y)p_k(Y). \tag{9}$$

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[1] We assume distributions are in the form of relative frequency histograms (as produced by simulation software). K is the number of categories, Δy is their width, M is the number of trials, f_k is the frequency count, and $p_k = f_k \cdot \Delta y / M$.

If $E\{v(Y)\} < 0$ then the decision maker expects a loss (on average). If $E\{v(Y)\} > 0$ then the decision maker expects a gain (on average). $E\{v(Y)\}$ is NOT equivalent to $E\{Y\}$ under any circumstances. Decision makers faced with risky decisions *do not exhibit risk neutrality* – a fact long established.

The use of this measure in the risk assessment decision problem is straight forward. If $E\{v(Y)\} \geq 0$, then we expect NO loss given the *status quo*. Everything is cool, don't worry be happy! If $E\{v(Y)\} < 0$ then we expect a loss. The *status quo* is "too risky" – something must be done (to reduce the risk)! We now face a risk management problem.

The answer to the risk management problem requires ordering all the alternatives by the risk measure (9). Now, however, the expected payoff is dependent on the alternative, A. We indicate this by replacing P_Y with $P_{Y|A}$ (i.e., the distribution of Y conditioned on the alternative A that we implement). Thus,

$$R_1(A) = E\{v(Y|A)\} = \sum_{k=1}^{k=K} v(Y)p_k(Y|A).$$

The risk minimizing alternative is $A = A^*$ where

$$R_1(A^*) = \min_A[R_1(A)] = \min_A[E\{v(Y|A)\}] = E\{v(Y|A^*)\}.$$

Of course, this ordering must be reconciled with the ordering induced by the cost. In most cases a trade-off must be made between the desire to minimize risk and to minimize cost (e.g., A^* is the most costly alternative while the least costly alternative reduces risk the least).

4.5.2 Probability of Loss

Research on decisions under risk finds decision makers are *loss averse* (as distinct from risk averse). Losses loom larger than gains in deciding what to do. Therefore it is natural to consider a second measure of risk that directly responds to this observation. One such measure is the probability of loss. For the preferences pictured in Figure 1 we have:

$$R_2 = \Pr\{v(Y) < 0\} = \sum_{k=1}^{k=K^R} p_k(v) = \Pr\{Y < Y_R\} = \sum_{k=1}^{k=K^R} p_k(Y). \tag{10}$$

where K^R denotes the category index closest to, or including Y^R. For the case pictured in Figure 2 we have:

$$R_2 = \Pr\{v(Y) < 0\} = \sum_{k=K^R}^{k=K} p_k(v) = \Pr\{Y < Y_R\} = \sum_{k=K^R}^{k=K} p_k(Y).$$

Note the computation of this measure is easier than the computation in (9) since we only need P_Y.

The use of this measure in risk assessment is similar to that outlined above. If $R_2 \leq R_{\max}$ where $R_{\max}$ is the decision maker's maximum acceptable level of risk then the *status quo* is NOT "too risky".– everything is OK, don't worry, be happy. If $R_2 > R_{\max}$ then the *status quo* is "too risky" and a risk management problem confronts us.

As above, we use (10) to induce an ordering over the risk management alternatives. This now requires computing

$$R_2(A) = \Pr\{v(Y|A) < 0\} = \sum_{k=1}^{k=M^R} p_k(Y|A)$$

for each alternative. The risk minimizing alternative is $A = A^*$ where

$$R_2(A^*) = \min_A[R_2(A)] = \min_A[\Pr\{v(Y|A) < 0\}].$$

Note that $R_1 = R_2$ when the decision maker exhibits *extreme loss aversion*. This happens when the decision maker prefers *never* to experience a loss. Such a preference is consistent with a payoff function where $v(Y) = -\infty$ for all Y corresponding to a loss, while $0 < v(Y) < \infty$ for all Y corresponding to gain. This form of the payoff function is denoted by $v_\infty(Y)$ and is defined by

$$v_\infty(Y) = \lim_{\lambda \to \infty} \frac{1}{\lambda} \left[\lim_{\substack{\beta \to 0 \\ \gamma \to 0}} v(Y) \right].$$

This limiting case is a payoff function that looks like a relay switch or step function with $v_\infty(Y) = 0$ for $Y < Y_R$ and $v_\infty(Y) = -\infty$ for $Y \geq Y_R$. This gives [see Wall (2010)]

$$R_2 = \Pr\{v_\infty(Y) < 0\} = \Pr\{Y < Y_R\} = \sum_{k=1}^{k=M^R} p_k(Y) = \sum_{k=1}^{k=M} v_\infty(Y)p_k(Y) = E\{v_\infty(Y)\} = R_1.$$

4.6   Summary

Risk assessment and risk management are decision problems. Their formulation requires definition of the three elements described so well by Ron Howard: (a) a set of alternatives; (b) an information structure connecting alternatives to the outcome of interest; and (c) preferences. The definition of risk popularized by K&G derives from an engineering safety context that assumes (a) and concentrates on development of (b). It does NOT say anything about (c). The K&G definition is motivated by the desire to understand a complex system, its possible failure modes and their probability of occurrence. This produces the information needed to relate the probable outcomes to the two alternatives: (i) NO there is no "significant" risk; or (ii) YES there is "too much risk" to ignore. Decision maker preferences are left out because the risk analysis that is the focus of K&G is a public policy issue and *the relevant preferences are not available to the analyst*. Only though political and institutional processes are these determined. The managerial perspective, and its definition of risk, is different from that of K&G. The important difference here is the preference information. The payoff function is known to the decision maker. It is the product of consideration of organizational goal(s) and objective(s) by a single person, or a relatively small group. This may take some deliberation to obtain agreement in a small group, but the result is still a payoff function. For the managerial decision problem the definition of risk becomes:

Risk is a *combination* of the answers to four questions:

(1) What can go wrong? ANSWER: F
(2) How likely is it to go wrong? ANSWER: P_F
(3) If it does go wrong, what are the outcomes? ANSWER: Y and P_Y
(4) How do you feel about it (the likely Y)? ANSWER: *preferences as expressed in* $v(Y)$

The original definition of risk as portrayed by the triple in (1) now has another form based on the system representation in (5):

$$\{Y, P_Y, v(Y)\}. \tag{11}$$

The the outcome of interest replaces the named scenario. The thing that "can go wrong" is Y taking on a value different from what the decision maker prefers (given what was assumed about F). The probability distribution of the Y replaces the probability distribution for F. "Consequence" no longer refers to the resulting outcome; it refers the preferences of the decision maker as described by $v(Y)$.

5 Example

Consider driving home after work on a Friday to pick up your date for an evening of dining, dancing and fun. The transit to your destination is fraught with hazards: traffic congestion, accidents, mechanical failures, personal injury, etc. You do not want to keep your date waiting (it can produce "bad" consequences). Since you lost your cell phone this morning, you can't communicate with your date once you've left the office (besides, you're not suppose to use the cell phone when you drive). So, what time do you tell your date to expect you?

The outcome of concern is $Y = T$, the time it takes to drive home. Is it reasonable to tell your spouse you will be home in 15 minutes, 30 minutes, 45 minutes? Let's look at how our two approaches to risk assessment would proceed.

5.1 The engineering safety approach

The engineering safety approach performs a fault analysis. First the normal state of the system is explicitly defined. In the example this is represented by the probability distribution of travel time given that no problem occurs with the auto. For example, if "all goes well" then travel time is a random variable with mean = 0.5 hr. Second, the ways in which the system fails to achieve normal operation is defined. For example, there could be: (a) mechanical failure; (b) traffic congestion; (c) an accident involving material damage; (d).an accident involving bodily injury; etc. Each of these "failure modes" would be decomposed into more detail. For instance, mechanical failure could result from a flat tire, loss of coolant, lack of petrol, ignition system failure, etc. There are many scenarios, one for each probable combination of events and referenced by a unique name. Third, each of the individual events comprising a scenario are assigned a probability. These individual probabilities are combined using the laws of probability and assigned to the respective named scenario. Fourth, each of the individual events in a scenario is evaluated in terms of the time it adds to the total travel. Finally these are summed and assigned to the named scenario.

To illustrate in more detail, suppose that traffic analysts decide to measure travel time, T, in hours and that data indicates the "normal" travel time is closely approximated by a PERT-distributed random variable with min = 0.25 hrs., max = 0.75 hrs. and most likely value = 0.5 hrs. Normal travel time is defined as travel time in the absence of any failure scenario.

Safety engineers define only three fault events, or failure scenarios that generate travel times longer than "normal": mechanical failure, M; traffic congestion, C; and accident, A. Suppose further analysis indicates the following scenarios form a "complete" set in the sense defined by Kaplan (1997):

- S_1 : M

- S_2 : C

- S_3 : A

Assume detailed analysis shows these occur with the following probabilities:

- $p_1 = p(M) = 0.03$

- $p_2 = p(C) = 0.10$

- $p_3 = p(A) = 0.02$

Finally, data analysis, reveals the total travel time, T_i, for each scenario is PERT-distributed with:

- T_1 : min = 0.5 hrs., max = 2.5 hrs., most likely = 1.0 hrs.

- T_2 : min = 0.75 hrs., max = 3.0 hrs., most likely = 1.25 hrs.

- T_3 : min = 0.5 hrs., max = 3.5 hrs., most likely = 2.5 hrs.

The results of such an analysis is summarized by a set of three triples contained in Table 1 :

TABLE 1: TRAVEL TIMES GIVEN PROBLEM EVENT

Scenario	Probability	Outcome
M	0.03	$PERT(0.5, 2.5, 1.0)$
C	0.10	$PERT(0.75, 3.0, 1.25)$
A	.02	$PERT(0.5, 3.5, 2.5)$

The reason for this structured approach is now clear. The second and third columns of this table define the probability distribution of the outcome of interest. This is a mixture distribution made up of the three PERT components and is pictured in Figure 3. This is the key information for the decision maker. Risk cannot be assessed, however, because without decision maker preferences (the payoff function) we cannot evaluate the implications of this information.
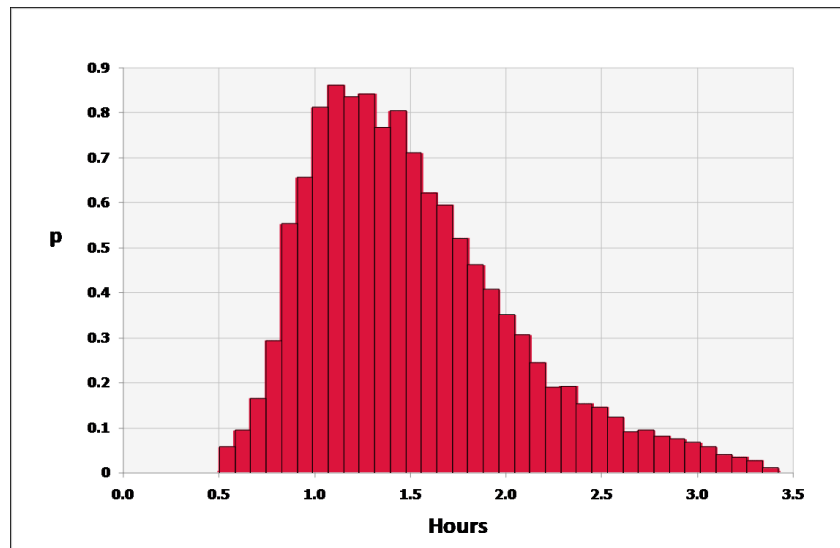

Figure 3: Travel Time Distribution given a "Failure" Occurs

5.2   The managerial decision context

Let us now recast this problem in the context of a managerial decision problem. The decision maker is our friend who's date must be notified when to expect to meet our friend for an evening on the town. The problem is to not keep the date waiting. Being early is no problem, but being late can ruin the evening. Our friend must commit to a pick-up time. At what time from now should it be: 15 min., 30 min., 45 min., an hour?

What can go wrong? The answer is obvious and stated in terms of the outcome of concern (the "consequence" in the language of K&G): you are later than you said you'd be. How likely is it to go wrong? The answer to this question is determined by your structural model. You know congestion could be a problem. You know mechanical failure could occur. You know there could be an accident. It is obvious to you that travel time, T, is a function all the possible situations:

$$T = f(C, M, A).$$

You also happen to have two very smart analysts working for you who are experts in modeling and data analysis. You put them on the case. A few minutes later they come back and say: "It's easy, all you need is simulation modeling". They say travel times have a mixture distribution composed of two distinct parts: (i) the first describes travel time when no problems arise (nothing breaks down, no traffic congestion, accidents, mechanical failure.); and (ii) the second describes travel times when there are problems. They have researched the data and find there are three events that cause problems (the same as determined in the engineering safety analysis). Their combined probability is $P_F = 0.13$, representing the likelihood of any failure/problem/difficulty. The likelihood of a "normal" commute home is $P_N = 1 - P_F = 0.87$. They also find that "normal" travel time is described by a PERT-distributed random variable with min = 0.25 hrs., max = 0.75 hrs. and most likely value = 0.5 hrs. (they have referenced the same traffic data as the safety engineers). Their simulation model describing travel time, T, is

$$T = Z \cdot T_N + (1 - Z) \cdot T_F \tag{12}$$

where T_N is "normal" travel time, T_F is travel time given a "failure" has occurred, and Z is a Bernoulli random variable with $p = P_N$. T_F is a random variable whose value is sampled from the distribution of T_1 with probability p_1, the distribution of T_2 with probability p_2 or the distribution of T_3 with probability p_3.

$$T_F = \text{mixture } \{T_1, p_1; T_2, p_2; T_3, p_3\}.$$

Each T_i is PERT-distributed as in Table 1 (again, the analysts have employed the same data as the safety engineers). Thus, the distribution of T_F is in Figure 3. This looks reasonable to you (equations were never your strength – but your "boys" are smart and know their stuff). You tell them to go ahead and get some results that you can understand!

The analysts return a minute later and show you the simulation output. It's the estimated probability distribution shown in Figure 4. You look at it and they explain its meaning. You understand now that travel time could be considerably longer than the 30 minutes you expect. What do you tell you date? Is it too risky to say you'll be home in half an hour?
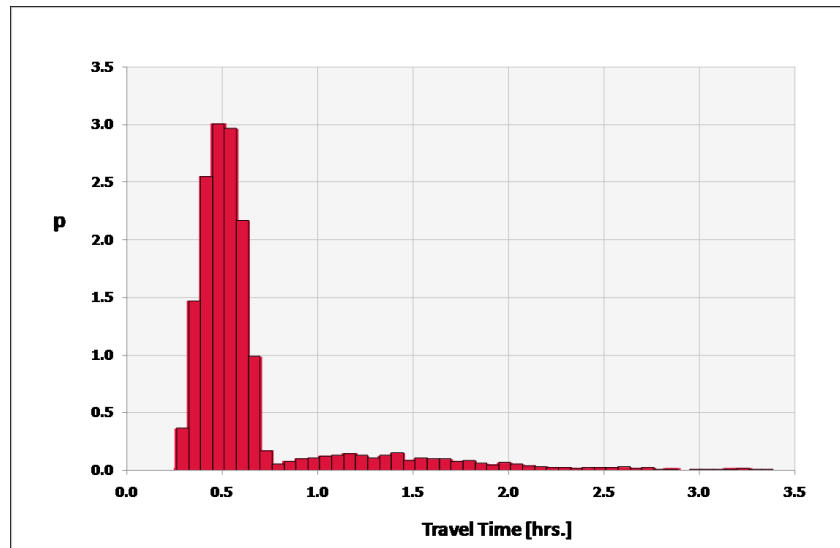

Figure 4: Simulated Travel Time

5.2.1 The (Extreme) Loss Averse Decision Maker

Suppose you know that if you arrive earlier than announced, it doesn't matter. Arriving on-time, five minutes early or 15 minutes early result in the same consequence – you wait for your date. On the other

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hand you know if you are later than announced there will be "trouble" – arriving later than announced always results in "trouble". You don't mind being early, but you never want to be late! You are *loss averse*. The distribution in Figure 4 tells you that

$$P(T > .50 \text{ hrs.}) = 0.573, P(T > 0.75 \text{ hrs.}) = 0.146, \text{ and } P(T > 1.0 \text{ hrs.}) = 0.126.$$

There is better than a one-in-two chance travel time will be more than 30 minutes. *Can you accept those odds?* If not then you'd decide on a later arrival time. If you can accept the odds implied by $P(T > 0.75$ hrs.$) = 0.145$, then you'd tell your spouse to expect you in 45 minutes. If not, then you consider the consequences for you of $P(T > 1.0 \text{ hrs.}) = 0.126$? If you can "live with" this prospect, then you'll call your spouse and say you'll be home in one hour.

The loss averse decision maker *cannot decide what to do without considering P_T in terms of the payoff function, $v(T)$.* In this instance, $v(T) = v_\infty(T)$ and is only a function of T_R, the reference point for travel time.

5.2.2 The Expected Consequences Decision Maker

Suppose you are motivated by expected payoff (consequence). You know two things concerning the consequences of your decision: (i) the later you arrive after your announced time, the worse things will be; and (ii) it doesn't matter how early you arrive –.it's all the same to your spouse. You just will have to wait until the spouse is ready.

The understanding date. You have a very understanding date who is well aware of the potential problems with Friday night traffic. Still, it is better to be on-time if at all possible. Your preferences in this case produce a payoff function like that depicted in Figure 5, where the payoff function intersects the T axis at $T = T_R$ (illustrated using $T_R = 1.0$). What would you decide if this was your payoff function? The probability distribution of arrival times is exactly as in Figure 4, but now deciding what T_R to use depends on $E\{v(T)\}$. What's best?
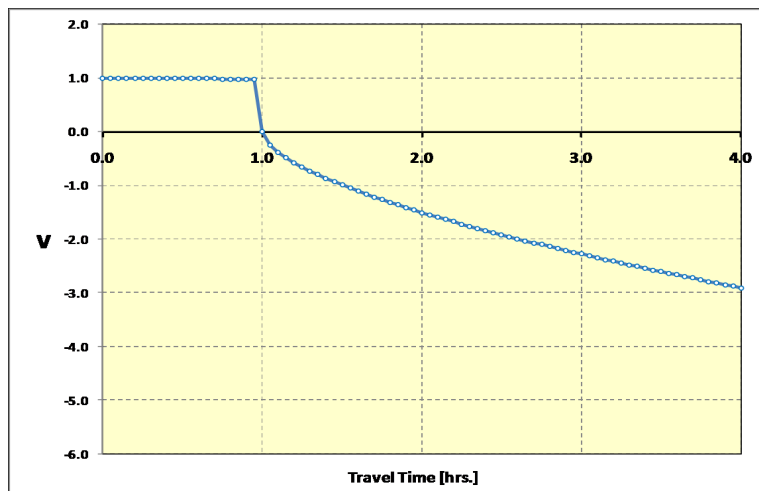

Figure 5: Payoff Function $v(T)$ with
$\beta = 0.01, \gamma = 0.60, \lambda = 1.5$

Simulation of (12) tells us the answer when you view Figure 4 in the light of Figure 5. The results are summarized in Table 2.

TABLE 2: PAYOFF SUMMARY

T_R	$P(v < 0)$	$E\{v(T)\}$	$P(T > T_R)$
0.25	1.000	$-.857$	1.000
0.50	0.574	0.058	0.574
0.75	0.146	0.659	0.146
1.00	0.126	0.734	0.126

All announced arrival times greater than or equal to 30 minutes produce positive expected values. If you arrive early all the "gains" are the same, so announcing you'll be home in half an hour is a solution.

The not-so-understanding date.   Your date definitely does not like to wait. If you say you'll be there in one hour, you better not be late! Your preferences in this case produce a payoff function like that in Figure 6 (again illustrated it with $T_R = 1.0$ hour). Now what would you decide with these preferences over T? What's best?
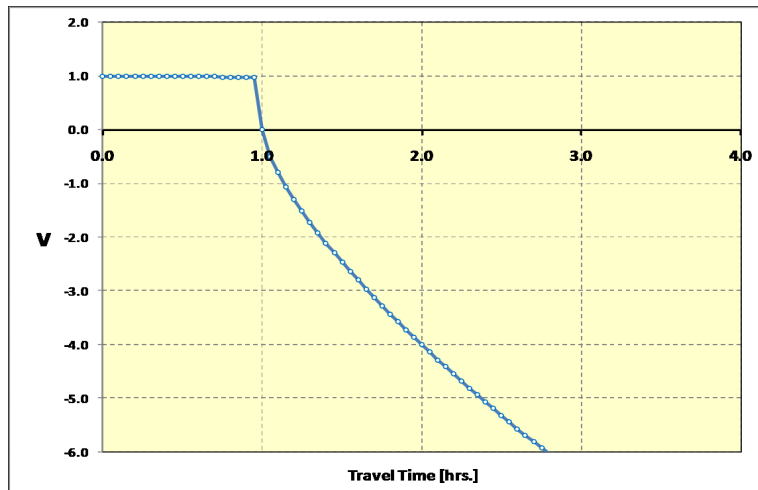

Figure 6: Payoff Function $v(T)$ with $\beta = 0.01, \gamma = 0.70, \lambda = 4.0$

Simulation again gives the answer by producing an estimation of the distribution of $v(T)$ from the distribution for T depicted in Figure 4. The results are summarized in Table 3. Now the only alternatives with expected "gain" are those with announced arrival times at least as great as 45 minutes. Anything less and you might a well forget about any fun tonight!

TABLE 3: PAYOFF SUMMARY

T_R	$P(v < 0)$	$E\{v(T)\}$	$P(T > T_R)$
0.25	1.000	-1.973	1.000
0.50	0.575	$-.432$	0.575
0.75	0.146	0.363	0.146
1.00	0.125	0.523	0.125

6   Conclusion

The context of the managerial decision problem provides a much richer and more specific information structure than that of the problem originally addressed by K&G. The outcome of interest and it's unit of

measurement are defined. This includes the equation specifying its values in terms of the variable(s) under the control of the decision maker, X, and the variable(s) beyond control (e.g., the future condition(s), F).

The answer to "What can go wrong?" is straight forward: the prediction of F is wrong (i.e., there is a prediction error). This leads to the "wrong" decision, X, and the "wrong" outcome, Y. Given eqs.(5) and (6), we find that the "wrong" F immediately expresses itself in terms of the "wrong" Y. Thus "What can go wrong?" can be answered in terms of the future conditions or *directly in terms of the outcome of concern.*

The same reasoning can be applied to the second question posed by K&G: "How likely is it to go wrong?" It is natural to state the answer in terms of the probability distribution over F, P_F. Again, however, use of eqs.(5) and (6) allows the answer to be stated *directly in terms of the probability distribution over* Y, P_Y.

While K&G's third question is now seen to be redundant, there still is need to ask a fourth question. The risk assessment/management decision problem is not complete until we have decision maker preferences. We need to ask: "How do you feel about it?" How do you, the decision maker, feel about the picture P_Y presents? We need to know the preferences of the decision maker for avoiding loss. We need to know how the decision maker values the probable outcomes depicted by P_Y. We need to know the decision maker's payoff function, $v(Y)$. This function will be reference-dependent with a reference value, Y_R, that separates those Y representing gain from those Y representing loss. It will be convex for losses and concave for gains. Moreover, it will exhibit loss aversion where losses loom larger than gains (where losses carry more weight than gqains).

We still find risk to be represented by a "triple", but with re-defined elements. *Risk is a combination* of three pieces of information:

$$\{Y, P_Y, v(Y)\}.$$

The precise combination depends upon whether the decision maker is motivated by maximizing $E\{v(Y)\}$ or minimizing $P\{v(Y) < 0\}$.

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