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Scale Invariance with Specific Application to IED Violence in Iraq 2003-2007

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Abstract

We explore whether violence in Iraq is gravitating towards a steady state level of violence observed in other civil wars, such as those in Colombia, Sri Lanka, Sierra Leone, among others. Using data from the Iraq Body Count for 2003-2007, we investigate whether civilian casualties follow a power law distribution. We replicate previous analyses of civilian casualties and confirm that casualties adhered to a power law distribution for 2003-2005. When we include data for 2006 and 2007, we find that the civilian casualties no longer followed a power law distribution. We argue that violence against civilians in Iraq remains chaotic.

Keywords: Iraq; Power Law; Kolmogrov-Smirnov; MLS;
JEL Codes: C22, H56, H77

INTRODUCTION

Casualty levels in wars tend to be power law distributed.[1] Doubling the severity of an event in terms of casualties decreases the frequency by a constant factor regardless of the size in question. Thus, there is no distinction between events at different scales and mass-casualty events occur more frequently than one might expect. Previous studies of civilian casualties in Iraq, for example, have shown the presence of a heavy tail, reflecting attacks with large numbers of casualties. The upsurge in violence in 2006, however, brings into question whether civilian casualties continue to adhere to a power law distribution or whether Iraq has entered a chaotic, and more dangerous, phase of civil conflict.

In this paper we explore whether violence in Iraq is gravitating towards a steady state level of violence observed in other civil wars, such as those in Colombia, Sri Lanka, Sierra Leone, among others. Violence in Iraq, in this case, would evolve towards a 'steady-state' where the likelihood of the severity of specific types of violence would be approximately the same as similar historical conflicts. Conditional on such an evolution, non-lethal alternatives, including economic development and political reconciliation, may prove to be more effective than lethal alternatives. Why? Non-lethal alternatives attempt to shift the entire distribution of violence downward while lethal alternatives, at least those practiced by democracies that are limited in terms of the application of violence, focus on segments of the violence distribution. Current efforts by the US to target Al Qaeda in Iraq, for example, aim to limit the tail of the distribution, that is, to mitigate or stop entirely the generation of catastrophic violence that seeks to further destabilize the Iraq state.

If, on the other hand, violence in Iraq is not evolving towards a historically observed steady state, the implications may be markedly different. Rather than attempting to shift the entire distribution of violence, a mixture of lethal and non-lethal means should be applied to different segments of the violence generation process. Economic and social incentives may decrease the relative wage of generating small-scale violence, while political and violence-generated incentives may serve to address larger-scale violence. In this case, the U.S. must also attempt to prevent the migration of those currently producing smaller-scale attacks to larger-scale attacks.

The remainder of this paper is structured as follows. In the next section, we briefly discuss power law distributions. We then review the literature on the distribution of violence in historical and current conflicts. After discussing the data, we examine the distribution of violence in Iraq from 2003 to 2007. Using publicly available data, we explore the inter-temporal evolution of the distribution of violence against civilians. The final section draws conclusions and offers recommendations as to the alternatives for the mitigation of violence in Iraq.

POWER LAW DISTRIBUTIONS

Power law distributions are used to describe the magnitude of events relative to their frequency of occurrence. Earthquakes, for example, are commonly cited as following a power law in that many have with no casualties; fewer have a small number of casualties, and it is relatively rare to observe an earthquake with a catastrophic number of deaths. Power laws appear widely in biology, demography, earth and planetary sciences, economics and finance, computer science, and physics, among other fields of study. The distributions of city size, academic citations, earthquake magnitude, webblog popularity, and income all appear to follow power laws.[2] Unlike a normal, or Gaussian distribution, the power law distribution implies that events with orders of magnitude larger or small than the mean are more common, that is, the majority of observations are not clustered around the mean of the distribution. Thus, a power law yields a finite probability to the occurrence of very large events that would be evaluated at a near-zero probability with the Gaussian distribution. As such, power laws are often referred to as scale invariant distributions.

As the number of deaths associated with violent conflict (x) can only take discrete values, we only consider the case of integer values with a probability distribution of the form $p(x) = \Pr(X = x) = Cx^{-\alpha}$, where α is constant parameter of the distribution known as the scaling parameter and C is a normalization constant. Both α and C are positive. A doubly logarithmic graph of the probability distribution is a straight line.[3] Once α is calculated, the constant c is determined by the condition that the sum of the distribution of $p(x)$ is 1.[4] The probability that a conflict has x or more victims is referred to as $P(X \geq x)$ and is the cumulative distribution obtained from $p(x)$. A graph of the cumulative distribution function $P(X \geq x)$ and $ln(x)$ will also be a straight line but with a negative slope of α-1.[5] Scale invariance implies that there may be no qualitative difference between events of different orders of magnitude. Furthermore, events may not be Gaussian distributed and events of orders magnitude larger (and smaller) than the mean may be relatively common.

With the ubiquity of power laws in conflict studies researchers have proposed a variety of mechanisms to explain the prevalence of the laws, arguing for a common generator across conflicts. Despite the lack of consensus as to why the distribution of casualties adheres to a power law distribution across space and time, the fact remains that the slopes of the logarithmic transform are surprisingly similar across conflicts. As such, we would expect violence in Iraq to gravitate towards a power law distribution.

A BRIEF REVIEW OF THE LITERATURE

Following Richardson's (1948) seminal work that a power law distribution best describes the pattern of human fatalities resulting from intra- or interstate conflict from 1820 to 1945, a number of authors have found that violence tends to conform to a power law distribution in a variety of conflicts. While the slopes of the power function may be dependent upon the form of violence (intra- or inter-state, global terrorism), Richardson's arguments tend to hold sway. In this section, we briefly review the literature on the level and distribution of violence and why violence resultant from insurgencies also appears to conform to a power law distribution.

Richardson (1948) argued that the normalized power curve for the internal conflict in Manchoukuo, China in 1932 approached 2.29. Manchoukuo is of interest in that the number of bandits rapidly increased from 100,000 in March to 210,000 as a result of the dissolution of the remnant of Chang Hsueh-Liang's army. The increase in the number of bandits suggests that former soldiers turned to violence for economic reasons. Yet, while the number of bandits more than doubled in a relatively short period of time, the distribution of violence still conformed to a power law distribution. Richardson argued that bandits operated in self-contained attack units that aggregated for the purpose of aggression. As such, the members of a specific cell may merge with other cells (coalesce) or breakup (fragment). Eventually, the cells achieve a steady state of size and force projection.

The Manchoukuo situation is startling in its similarity to Iraq following the dissolution of the Iraqi Army in May 2003. On May 15th of that year, a day after arriving in Baghdad to assume the leadership of the Coalition Provisional Authority, Ambassador Paul Bremer announced CPA Order Number 1, followed on May 16th by CPA Order Number 2. CPA Order Number 1 effectively prohibited members of the Ba'ath party from holding government positions. CPA Order Number 2 disbanded the Iraqi Army. Regardless of who promulgated these orders, the effect was immediate. Hundreds of thousands of pre-dominantly Sunni officials and officers were removed from public service, without compensation or recourse. Whether the orders originated in Department of Defense or the White House, the widely acknowledged impact was to create a disaffected pool of Sunnis who formed the core of the initial insurgency.[6] If Iraq is similar to previous conflicts, we should observe a power law distribution of casualties similar to what Richardson observed in Manchoukuo.

More recent analyses suggest that other dimensions of violent conflict adhere to a power law distribution. The cumulative frequency distribution of the severity of interstate wars from 1820 to 1997 appears to follow a power law distribution.[7] A ten-fold increase in war severity decreases the probability of war by a factor of 2.6. Terrorist attacks worldwide since 1968 are also scale invariant.[8] Scale invariance appears to persist even when the level of economic development of the target country, the type of weapon used, and shorter time scaled are employed as controls. Casualties in intra-state conflicts in Iraq and Colombia and non-G7 global terrorism also appear to converge to a power law distribution.[9] Fatalities per incident in Iraq appear to be distributed in a similar fashion to those resulting from non-G7 terrorism, suggesting that violence in both cases is characterized by a large number of low-intensity events.[10] Furthermore, scale invariance persists across different weapon types, suggesting that violence in Iraq is decentralized and results from a myriad of actors. Not only does the literature suggest that violence conforms to a power law distribution, but some have gone so far to argue that the conflicts in Iraq, Afghanistan, Colombia, Indonesia, Israel, and Ireland are all converging to a universal distribution with a slope of 2.5.[11]

A variety of theories exist as to why violence appears to conform to a power law distribution. Violence may evolve towards a scale free steady state through self-organized criticality (SOC). Innovations in technology and infrastructure influence the pattern of conflict, increasing (decreasing) the area over which violent actors exercise power.[12] As evidenced by the impact of Improved Explosive Devices (IEDs) in Iraq, there is a continual cycle of innovation and counter-innovation. Even though U.S. forces have improved tactics, techniques, and procedures such that the number of IED events required to produce one fatality increased from 1 in 2004 to 6 in 2007, the number of attacks has also increased dramatically.[13]

Another line of reasoning argues that while individual insurgent cells continuously fragment and coalesce, the overall distribution of cells appears to be similar across conflicts, resulting in a steady-state level of violence generation.[14] From the perspective of this paper, this result in interesting in light of the fact that Iraq and Colombia have markedly different starting points. Violence in Iraq resulted initially from an inter-state conflict, which, after the collapse of the Iraqi state, evolved into an insurgency. The Iraqi insurgencies (as there is not a homogenous insurgency) continue to fragment and coalesce. Colombia, on the other hand, began as a small-unit insurgency, growing from approximately 200 fighters in 1996 to over 18,000 fighters within a relatively homogenous, hierarchical structure by 2005.[15] Even though the two intra-state conflicts have dissimilar origins, the underlying structure appears to be analogous.

Regardless of the mechanism by which casualties are generated, the consensus in the literature is that scale free power laws best approximate the organization of conflicts. Furthermore, there is wide agreement that conflicts dynamically evolve to a steady-state level of conflict. We should thus observe in Iraq a similar evolution of casualties towards the historically generated norm. We turn to this question by first discussing the data employed in the analysis in the next section.

IRAQI CIVILIAN CASUALTY DATA

We employ casualty data from the Iraq Body Count (IBC) for the purposes of this study. The IBC project attempts to establish an independent and comprehensive public database of civilian deaths in Iraq derived from media reports. The IBC requires two or more media reports of the same incident after January 1, 2003 for inclusion in its database. The IBC data reports civilian status, minimum and maximum number of reported deaths, date, time, location, type of target, and weapon. Through December 31, 2007, the IBC reports that a minimum of 81,020 Iraqis have died since January 1, 2003 from violent attacks.[16]

Some critics may point to the reliance on media reporting to cast doubt on the efficacy of the IBC data. Faced with the nearly impossible task of creating a tally of civilian casualties in a dangerous environment, the IBC developed a small set of standard variables that could be extracted from media reports in a consistent and transparent manner (Fischhoff, Atran, and Fischhoff, 2007). The reported number in casualties contained in the IBC data may underreport actual casualty data. The United Nations Assistance Mission for Iraq reported a total of 34,452 violent civilian deaths in 2006[17], approximately 9,000 more deaths than the minimum number reported in the IBC data. Burnham, Lafta, Doocy, and Roberts (2006) argue that as of July 2006 that there were 601,027 excess Iraqi deaths due to violence.[18] Opinion Research Business reported in excess of 1 million deaths in their September 2007 survey of 1,499 adults.[19] Furthermore, there may also be an incentive for the underreporting of casualty data to promote the appearance of improvements in security.[20] It would thus appear that the IBC data, by relying solely on two or more media reports of deaths from violence, serves as a lower bound on the estimates of Iraqi civilian casualties.

For the purposes of our analysis, we employ casualty data arising from specific incidents and thus exclude aggregated casualties reported, in most instances, by the Baghdad city morgue. As we are unable to ascertain the discrete events that led to these observations, to include them would undoubtedly overstate the likelihood of mass casualty events. This approach mirrors that of previous analyses. When we exclude these observations, the number of incidents falls from 13,569 with 81,020 casualties to 13,452 with 55,050 casualties.[21] We employ this data for the remainder of our analysis.

While conventional wisdom suggests that violence abated in 2007, our analysis suggests that violence continued to increase, albeit at a lower annual rate from 2003-2006 (Table 1,2,3). The number of incidents increased significantly in magnitude in 2006,

[TABLE 1,2,3]

reflecting the sectarian civil war that engulfed Baghdad and other mixed municipalities. The increase in U.S. combat troops appears to have restrained the growth in the number of reported incidents in 2007. Two questions emerge from these data: (1) Is this trend sustainable and (2) has the violence against civilian targets evolved since 2003? We argue in this paper that by examining the distribution of violence, we can make inferences as to the sustainability of the reduction of the rate of growth of violence against civilians in Iraq. We turn to this question in the next section.

THE DISRIBUTION OF VIOLENCE AGAINST CIVILIANS IN IRAQ

Examining the distribution of violent attacks resulting in deaths of civilians in Iraq provides information as to whether the violence is chaotic or whether the conflict is evolving toward a steady-steady observed in other conflicts. Previous research suggests that the cumulative distribution of violence satisfies a power-law relationship over a relatively large range. Given the escalation in violence in 2006 and 2007, we ask whether violence in Iraq against civilians continues to conform to the power law distribution.

Casualties per event in chronological order are illustrated in a semi-log plot in

[FIGURE 1]

Figure 1. A linear trend line fitted to the data suggests that the average number of casualties per incident is decreasing throughout the 2003-2007 period. Let $p(x)$ be the distribution of incidents with severity (number of casualties) x and let $P(X \geq x)$ be the complementary cumulative distribution obtained from $p(x)$. Examining the frequency

and the resulting cumulative distribution of incident severity suggests that violence in Iraq follows a power law distribution (Figure 2).

[FIGURE 2]

Using double logarithmic scales, we plot the cumulative frequency as a function of the severity of attacks against Iraqi civilians (Figure 3).

[FIGURE 3]

If we assume that events are independently and identically distributed, we may model the distribution of violence as a power law with exponent α, where the scaling behavior holds only for values at least some lower-bound $x_{\min}$. Initially, we assume that a power law holds for all the observed incidents, that is, $x_{\min} \geq 1$. Using this assumption, we find that the simple linear bi-variate regression of log $P(X \geq x)$ on log severity x explains approximately 94% of the variation in the cumulative frequency distribution.[22] We caution, however, that observation of Figure 3 suggests that a linear relationship is not as apparent as previous analyses in the literature. The slope estimate (-1.6) implies that a 10-fold increase in incident severity decreases the probability of an incident by a factor of 39.8 ($1/10^{-1.61}$), which is well outside the expected range.[23]

A potential criticism of this approach is that we assume $x_{\min}$ to be one. We then assume that $x_{\min}$ is equal to that previously observed in the literature. If casualties continued to conform to a power law distribution, this would be a reasonable approach. Setting $x_{\min} \geq 3$, for example, we find that the simple linear bi-variate regression of log $P(X \geq x)$ on log severity x explains approximately 96% of the variation in the cumulative frequency distribution (Figure 4).[24]

[FIGURE 4]

The slope estimate increases to -1.70, suggesting that a ten-fold increase in incident severity decreases the probability of observing an incident fifty fold.[25] We obtain similar results for $2 \leq x_{\min} \geq 8$. This evidence suggests that the previous findings of violence in Iraq conforming to a power law distribution may be dependent to the period of analysis.

While the preceding analysis is suggestive, we note the literature casts doubt on the efficacy of using linear regression analysis to determine α.[26] We proceed as follows. We construct estimates for the power law exponent α for the entire period of analysis, 2003-2007, and then examine a rolling window starting with 2003 and adding an additional year of data to explore whether the war in Iraq adheres to the power law over time. We employ a Kolmogorov-Smirnov goodness-of-fit test to select $x_{\min}$ The KS test statistic is based on the test statistic:

$$KS = Max_{x \geq x_{\min}} [S(X) - P(X)] \tag{2}$$

Where $S(x)$ is the hypothesized cumulative distribution function using the estimated power law exponent α and $P(x)$ is the empirical data based on the observed data. We employ x_{min} equal to the minimum value of x where we cannot reject the hypothesis that the data beyond x_{min} satisfies the Kolmogrov-Smirnov (KS) goodness of fit test at the 95% confidence level. We estimate the power law exponent with the Maximum Likelihood Estimator by[27]:

$$\alpha = 1 + n \left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}} \right]^{-1} \tag{1}$$

where x_i , $i=1 \ldots n$ are the observed values of x such that $x_i \geq x_{min}$. We estimate the expected statistical error σ on α as:

$$\sigma = \sqrt{n}\left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}\right]^{-1} = \frac{\alpha - 1}{\sqrt{n}} \tag{3}$$

We then estimate the *p*-value as the fraction of KS statistics for the resampled data sets whose value exceeds the KS statistic for the real data. If the *p*-value is sufficiently small, the power law distribution is ruled out.[28]

For 2003, we estimate that α=1.6 ($x_{min} \geq 1$, p_{ks}= 0) (Figure 5). For 2003-2004, we estimate that α=2.26 ($x_{min} \geq 8$, p_{ks}=0.94) (Figure 6). For 2003-2005, we estimate that α=2.28 ($x_{min} \geq 1$, p_{ks}= 0.90) (Figure 7).

[FIGURE 5,6,7]

Civilian casualties through 2005, as noted previously in the literature, do adhere to a power law distribution. We thus replicate the results of Johnson et. al. (2007) and Alvarez-Ramiez et. al (2007). The estimated exponent also approaches the previously observed 2.5, which is the exponent for non-G7 global terrorism.

When we include 2006, however, we estimate that for 2003-2006 that α=2.23 ($x_{min} \geq 4$, p_{ks}=0.00) (Figure 8).

[FIGURE 8]

Civilian casualties in Iraq no longer conform to a power law distribution. We note that 2006 observed a four-fold increase in violence, suggesting that Iraq became chaotic, with violence begetting more violence. At this point, non-lethal alternatives would, in essence, become ineffective, as establishing security would be paramount. If security were to be effectively established through the addition of combat forces, we would observe a return to the previously observed power law distribution and, perhaps, mitigation in the levels of violence directed against civilians.

For 2003-2007, we estimate that α=2.13 ($x_{min} \geq 3$, p_{ks}=0.00) (Figure 9). Contrary to the previous results in the literature; civilian casualties do not appear to conform to a power law distribution for the period of analysis. We examine whether 2006-2007 conforms to a power law distribution and find that it does not with α=2.14 ($x_{min} \geq 3$, p_{ks}=_0) (Figure 10). We also explore whether 2007 by itself conform to a power law distribution and find that α=2.13 ($x_{min} \geq 3$, p_{ks}=0) (Figure 11).

[FIGURE 9,10,11]

In summary, we find that civilian casualties did conform to a power law distribution from 2003 to 2005. The significant increase in violence and the shift in violence to larger scale attacks undermined this relationship. We find that violence from 2003-2007 and 2006-2007 does not conform to a power law relationship. This result suggests that the 'surge' of additional U.S. combat brigades in 2006 did mitigate the rapid increase in civilian casualties. Violence against civilians in Iraq, however, currently does not appear to conform to previously observed steady states, suggesting that the environment remains chaotic.

THOUGHTS

Prior to the bombing of the Golden Mosque in Samara, violence in Iraq appeared to conform to a power law distribution. After this point, we believe, violence in Iraq became chaotic, that is, violence begat increasing violence. In effect, a new conflict emerged, a civil war that not only increased violence between Sunni and Shia but also fostering intra-Sunni, intra-Shia, and anti-occupation violence. We argue that 2006 observed the rapid emergence and escalation of several civil wars within Iraq, some of which remain to this today. In this section, we discuss the stylized facts that provide credence that Iraq became and remains chaotic in 2006.

First, we observe that the number of attacks against Iraqis (civilian and military) and U.S. forces significantly increased in 2006. Thus, while we observe that the effectiveness of IEDs significantly declined in 2006, requiring more IEDs to generate the same amount of U.S. casualties, the absolute number of attacks increased as well.[29] Yet, an increase in the number of violent attacks would not, per se, destabilize the power law relationship. If violence were distributed equally, that is, if the distribution was maintained but at a higher magnitude, then violence in Iraq would have continued to conform to a power law distribution. What occurred, on the other hand, was a shift in the composition of violence such that the likelihood of higher casualty attacks increased relative to lower casualty attacks.

Second, the stated objective of the Samara attack and other mass casualty attacks in 2006 was to provoke further violence. Al-Qadea in Iraq and other insurgent groups stated intent is to destabilize the Iraqi government through indiscriminate violence. Shia militias responded in kind, creating an endogenous cycle of violence. Simultaneously, within group violence increased as Sunni militias turned against Al-Qadea (known as the Anbar awakening) and the Sadar and Badr militias competed for power. The addition of U.S. forces and the shift towards decentralization combat operations (as opposed to centralizing U.S. forces in Forward Operating Bases) reduced the rate of increase in the level of violence in 2007, however, whether this reduction is sustainable remains an unanswered question. If the additional U.S. combat brigades affected the distribution of violence in Iraq, then we would expect that violence would conform again to a power law distribution. Sadly, this does not yet appear to be the case.

Third, as noted early in this paper, Iraq may be a realization of a new form of warfare, that is, fourth-generation warfare. In Iraq, we observe a confluence of technologies unobserved in previous conflicts. Information warfare is an import ant component of each member's campaign strategies. Weapons technology is decentralized and amplified such that insurgent groups can defeat all of the land-based weapons systems of the U.S. on a singular basis. Couple the emergence of these tactics in a heterogeneous society with strong economic incentives for violence and a history of violence yields a complex operational environment. One might argue that violence remains the same, but the speed at which information influences the level and distribution of violence is unparalleled in human history.

We suspect that the confluence of events provided the catalyst for the increase in violence in 2006 and sustainment of violence into and through 2007. While civilian casualties in Iraq prior to 2006 conformed well to a power law distribution, this is no longer the case. This finding suggests that previous arguments that Iraq shares common patterns with non-G7 terrorism are in error. While civilian casualties in Iraq as a whole do not conform to a power law distribution, we question whether this finding is extendable to all provinces. This result may be driven by violence in the provinces surrounding and including Baghdad. Obviously, violence is not homogenously distributed across Iraq, suggesting that some provinces may be stable, other chaotic. We leave this question for future research.

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[1] Richardson (1948) and Richardson (1960).

[2] Newman (2005). Shirky (2003). Redner (2005).

[3] A straight line is a necessary but not sufficient condition in that variables governed by different data generating distributions may mimic the behavior of the power law distribution. An exponentially distributed variable with a limited number of observations, for example, would appear linear with doubly logarithmic axes.

[4] As such, c is a placeholder and is of no importance. The slope of the resulting line is $-\alpha$.

[5] Refer to Mitzenmacher (2004) and Newman (2005) for a detailed discussion regarding the application and derivation of power laws.

[6] There continues to this day to be a great deal of confusion as to who actually ordered these actions. Bremer argues that Douglas Feith, undersecretary of defense for policy, provided him the orders at the behest of Paul Wolfowitz, the deputy secretary of defense, who acted for Secretary of Defense Donald Rumsfeld. Bob Woodward, in State of Denial: Bush at War, Part III, quotes Rumsfeld as saying that the order came from elsewhere.

[7] Cederman (2003).

[8] Clauset and Young (2007).

[9] Johnson et al. (2005) adapting Eguiluz-Zimmerman dynamic equilibrium model of herding behavior in the stock market to examine the scale invariance of these conflicts. See also Eguiluz and Zimmerman (2000).

[10] Alvarez-Ramirez et al. (2007).

[11] Johnson, et al. (2005).

[12] Cederman (2003).

[13] Montgomery C. Meigs, director, Joint IED Defeat Organization, interview by Atkinson, Rick, "Left of the Boom", Washington Post, September 30, 2007

[14] Johnson et al. (2005, 2006) argue that a conflict characterized by the ongoing process of coalescence and fragmentation of attack unit will, in the long run steady state limit, have an exponent of 2.5 for the power function.

[15] Heiberg et al. (2007).

[16] See Iraqi Body Count, Available at: http://www.iraqbodycount.org/about/methods/ for a discussion of the data sources and methods of data extraction of civilian casualty data.

[17] UN Assistance Mission for Iraq (UNAMI) (2006), Human Rights Report. Available at: http://www.uniraq.org/aboutus/HR.asp, last accessed 5 November 2007.

[18] A cross-sectional cluster sample survey, Prof Gilbert Burnham Prof Riyadh Lafta ,Shannon Doocy and Les Roberts The Lancet 2006; 368:1421-14 Unlike the IBC data that replies on media reports, Burnham et al (2006) conducted a national cross-sectional cluster sample survey of mortality in Iraq. 50 clusters were randomly selected from 16 Governorates, with every cluster consisting of 40 households. This approach is still subject to a great deal of debate as to its legitimacy.

[19] The ORB survey asked "how many members of your household, if any, has died as a result of the conflict in Iraq since 2003?" with 78% of respondents answering none and the remaining respondents answering 1 or more. ORB estimates from the 2005 census of 4,050,597 households that this results in a confidence interval of 733,158 to 1,446,063 deaths from violence since 2003. Available at: http://www.opinion.co.uk/Newsroom_details.aspx?NewsId=78, last accessed 5 November 2007.

[20] Dagher, Sam, (2007), In Iraq, death tolls often in dispute, The Christian Science Monitor, August 3rd, 2003, Available at: http://www.csmonitor.com/2007/0803/p07s02-woiq.html, Last Accessed 5 November 2007.

[21] Of the 119 excluded observations, the average number of reported casualties was 218.6 with the median number of casualties equal to 58. The vast majority of these deaths resulted from small arms fire and, according to the IBC, "excludes trauma deaths from accidents."

[22] The estimated t-statistics for the intercept and independent variable are 8.86 and -41.17, respectively.

[23] Previous studies in the literature, including Alvarez-Ramizez et.al (2006), have excluded the period prior to the end of major combat operations on May 1, 2003. We include these observations as germane to the analysis of the level and distribution of violence in Iraq. With $x_{min} \geq 1$, the estimated slope parameter excluding observations prior to May 1, 2003 is 1.64 with an R-squared of .94. These regressions are available upon request.

[24] The estimated t-statistics for the intercept and independent variable are 8.86 and -41.17, respectively.

[25] With $x_{min} = 3$, the estimated slope parameter excluding observations prior to May 1, 2003 is 1.78 with an R-squared of .96. These regressions are available upon request.

[26] Clauset, Shalizi, and Newman (2007) argue that performing a least squares linear regression on the logarithm of the histogram shows significant biases under relatively common conditions. The results of these regressions may be suggestive but cannot be trusted. Goldstein, Morries, and Yen (2004) also argue that using MLE is far more robust than log-log plots or linear regression.

[27] Newman (2005) details the derivation of the formulas used to calculate α and σ by using the transformation method of converting a uniform distribution into a continuous power law distribution.

[28] Clauset, Shalizi, and Newman (2007)

[29] Montgomery C. Meigs, director, Joint IED Defeat Organization, interview by Atkinson, Rick, "Left of the Boom", Washington Post, September 30, 2007

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Figure 1
Log Plot of Fatalities Per Event

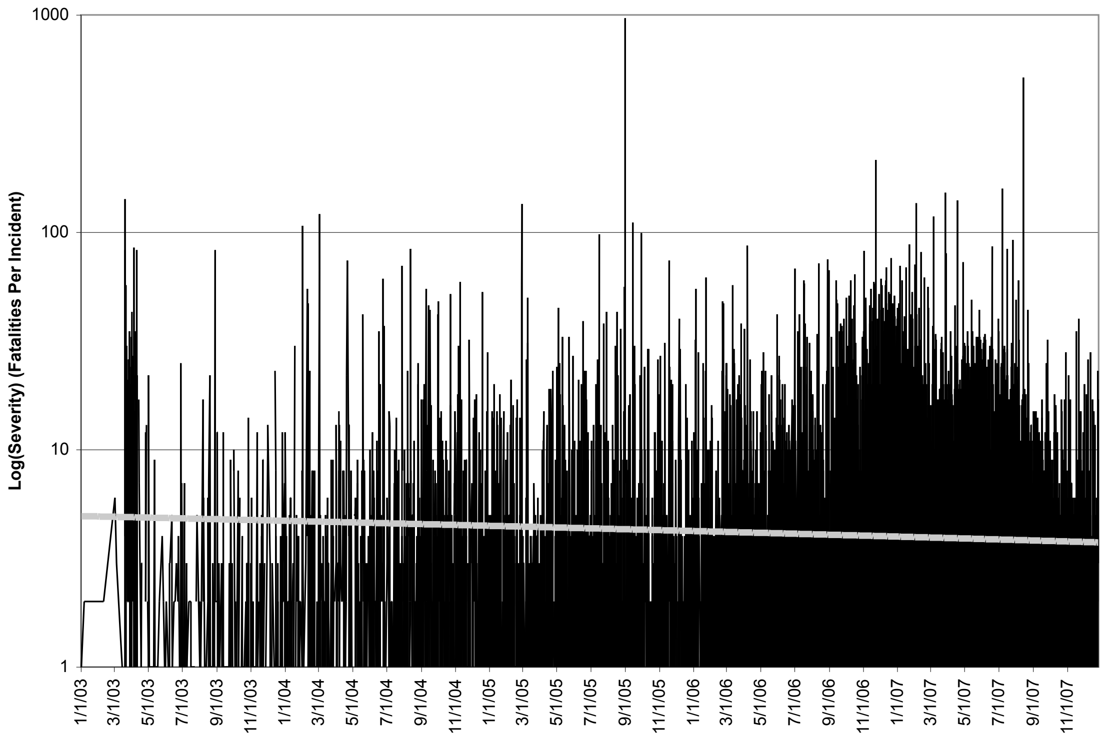

Figure 2
Frequency and Cumulative Distributions of Incident Severity

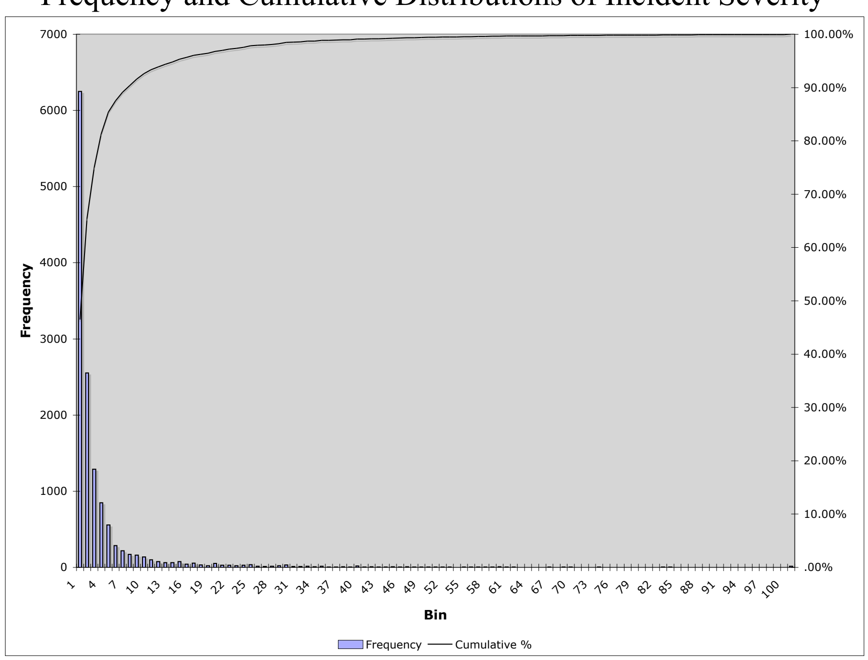

Figure 3
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
2003-2007

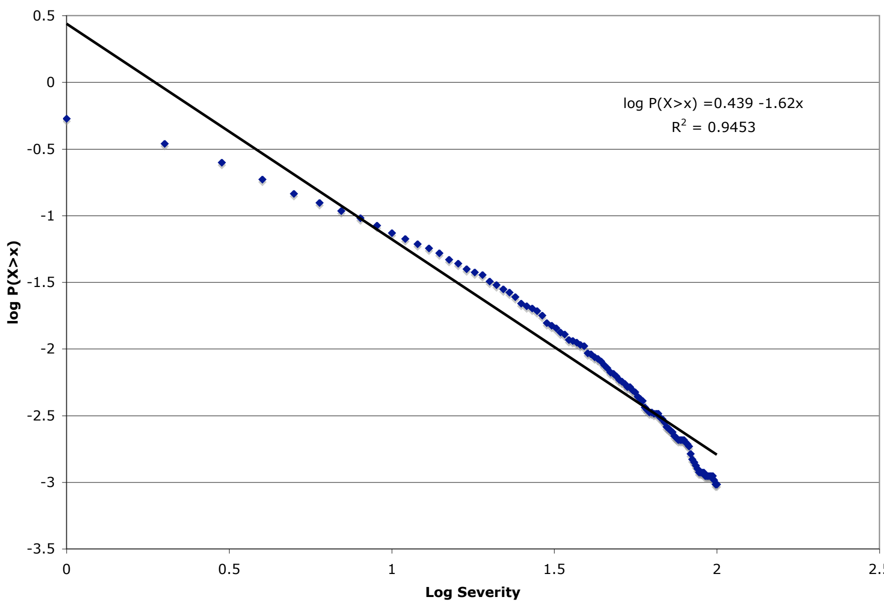

Figure 4
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties (Xmin = 3)
2003-2007

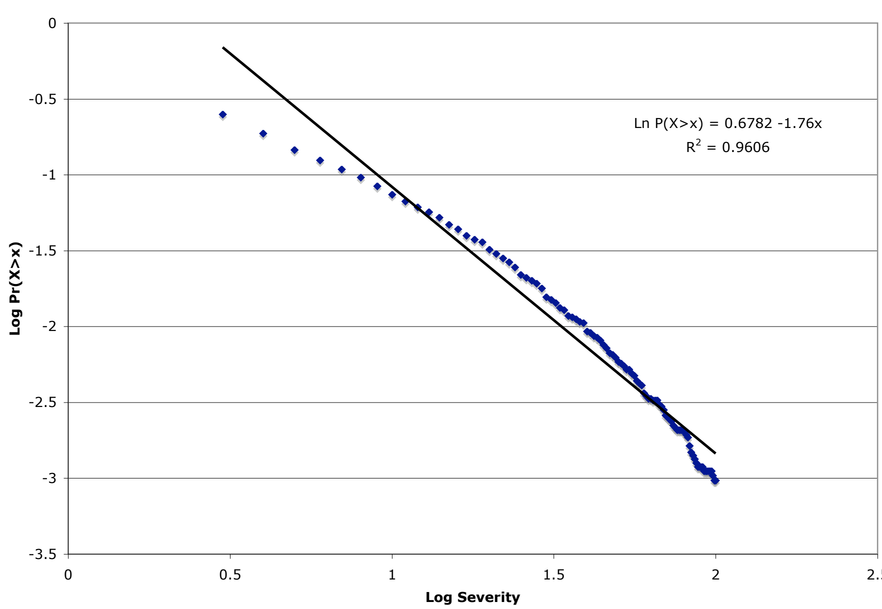

Figure 5
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} = 1) (α=1.66) (p_{ks} = 0)) (MLE)
2003

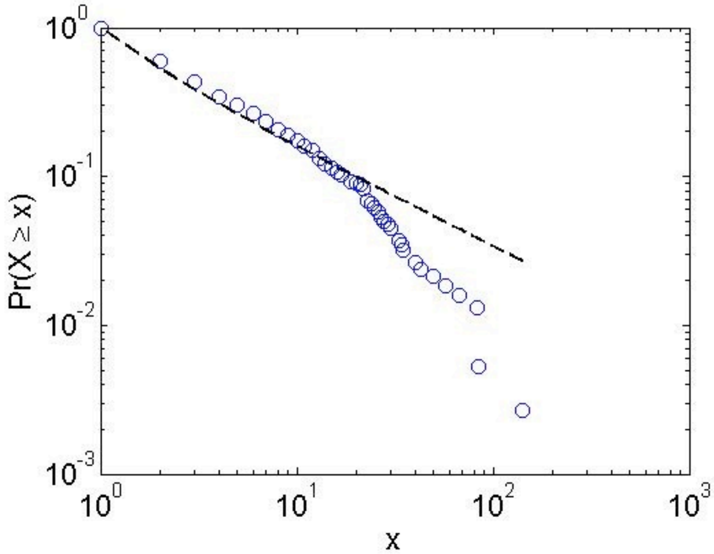

Figure 6
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} = 8) (α=2.26) (p_{ks} = 0.94) (MLE)
2003-2004

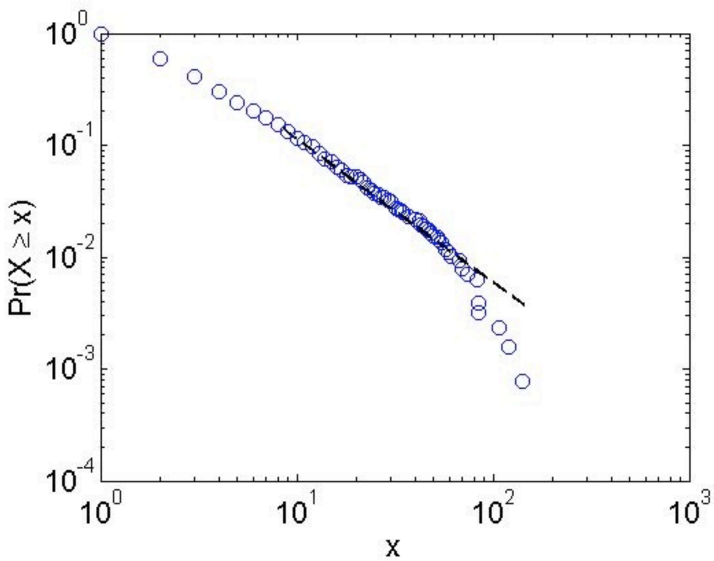

Figure 7
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} =8) (α=2.28) (p_{ks} = .90) (MLE)
2003-2005

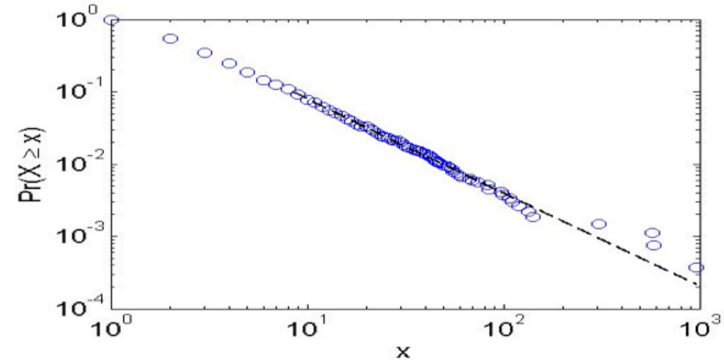

Figure 8
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} =4) (α=2.23) (p_{ks} = 0.001) (MLE)
2003-2006

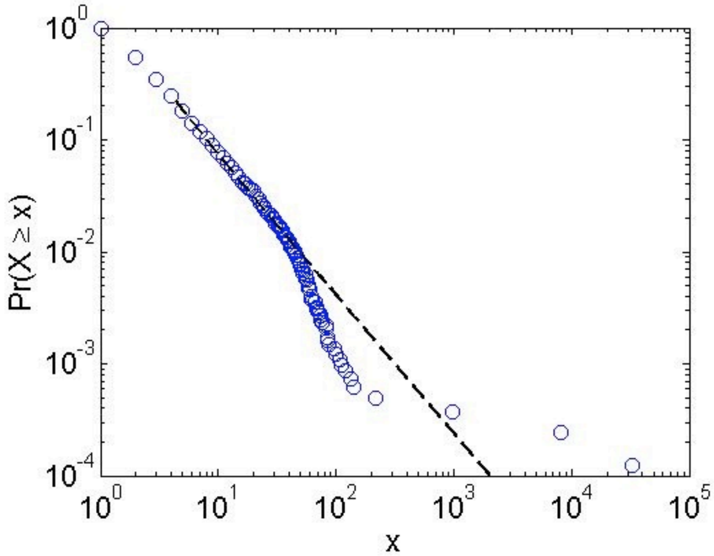

Figure 9
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} =3) (α=2.13) (p_{ks} =0) (MLE)
2003-2007

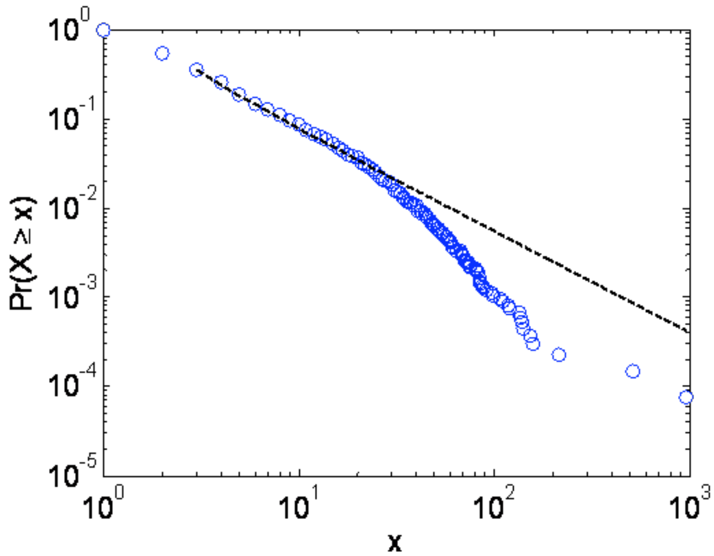

Figure 10
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} =3) (α=2.13) (p_{ks} =0) (MLE)
2006-2007

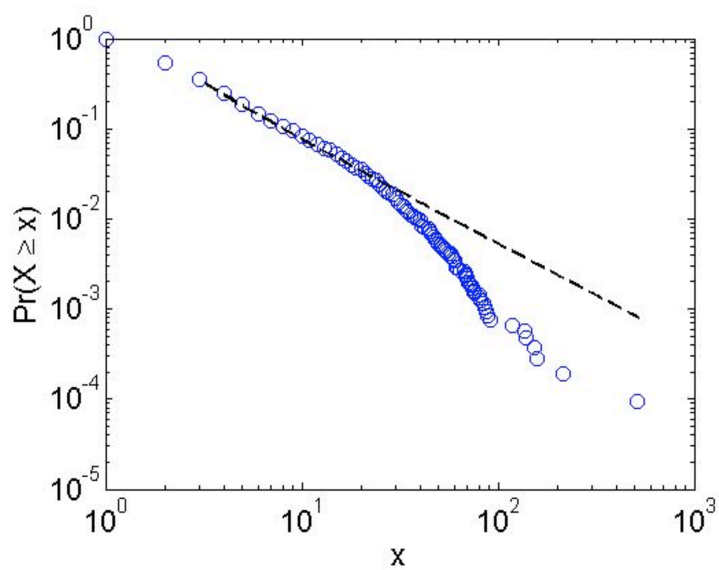

Figure 11
Log-Log Plot of the Cumulative Frequency Distribution of Severity
Of Civilian Casualties
(X_{min} =3) (α=2.13) (p_{ks} =0) (MLE)
2007

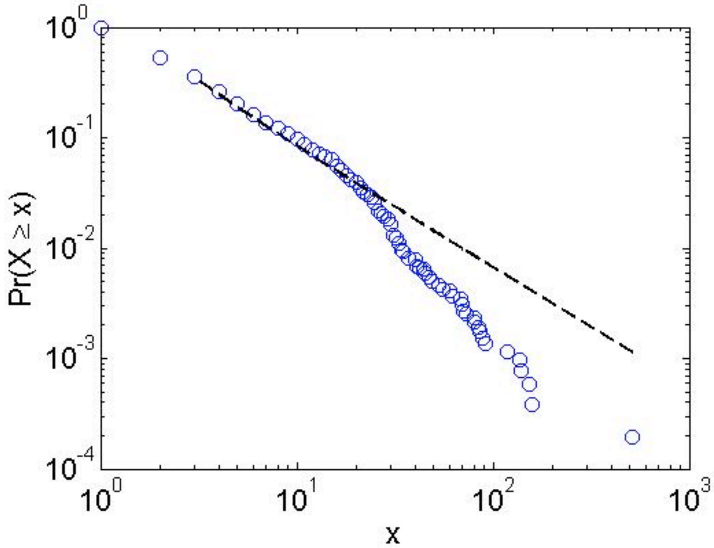

Table 1
Casualty Data
2003-2007

	IBC Database	Excluded Observations	Analytical Database
Number of Incidents	13,569	118	13,451
Total Number of Reported Deaths	81,020	25,987	55,033
Average Number of Deaths per Incident	5.97	220.23	4.09
Minimum Reported Deaths	0	0	0
Maximum Reported Deaths	1,473	1,473	965
Standard Deviation	33.77	259.99	12.46

Table 2
Casualty Data
Excluding Aggregated Incidents
2003-2007

	2003	2004	2005	2006	2007
Number of Incidents	395	902	1856	5074	5224
Total Number of Reported Deaths	2535	4130	7477	19063	21828
Average Deaths per Incident	6.41	4.58	4.02	3.75	4.18
Standard Deviation	13.46	9.80	23.52	7.61	10.78

We exclude reports that are aggregations of specific attacks. As illustrated in Table 1, these observations (typically from a municipal morgue) represent a large number of deaths from violence, typically for a week or month. Including these observations would overstate the likelihood of a mass casualty attack.

Table 3
Histogram of Violence Attacks Against Civilians in Iraq, 2003-2007

	2003	2004	2005	2006	2007	**Totals**
0	18	6	12	38	56	**130**
1 to 5	277	741	1622	4381	4335	**11356**
6 to 10	40	82	108	347	389	**966**
11 to 20	27	44	69	163	263	**566**
21 to 30	19	8	24	59	112	**222**
31 to 40	5	3	9	36	33	**86**
41 to 50	2	6	4	21	13	**46**
51 to 60	1	5	2	17	5	**30**
61 to 70	1	3	0	6	4	**14**
71 to 80	0	1	1	3	3	**8**
81 to 90	4	1	0	2	4	**11**
91 to 100	0	0	1	0	1	**3**
101 to 125	0	2	1	0	1	**4**
126 to 150	1	0	1	0	2	**4**
151 to 175	0	0	0	0	2	**2**
176 to 200	0	0	0	0	0	**0**
201 to 300	0	0	0	0	0	**1**
301 to 400	0	0	0	1	0	**0**
401 to 500	0	0	0	0	0	**0**
501 to 1000	0	0	1	0	1	**2**
Totals	**395**	**902**	**1856**	**5074**	**5224**	**13451**